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Uncertainty in an Interconnected Financial System, Contagion, and Market Freezes

This paper studies contagion and market freezes caused by uncertainty in financial network structures. Our model demonstrates that in a financial system where financial institutions are interconnected, a negative shock to an individual financial institution could spread to other institutions, causing market freezes because of creditors' uncertainty about the financial network structure. Our model also reveals that when both a large creditor and a continuum of small creditors are present, the size of the large creditor will affect the severity of market freezes substantially. Moreover, our model is used to examine central bank policies to alleviate market freezes.

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An important contagion mechanism emerged in the recent subprime mortgage crisis: with the rapid expansion of credit derivatives, financial institutions are interconnected through an increasingly complicated financial network. Consequently, when one of the financial institutions goes bankrupt, general uncertainty about the losses of other financial institutions in the network arises. This is because the complexity of the financial network makes it difficult for market participants to assess these losses. As a result, market participants can stop lending to one another; in other words, financial markets freeze. This contagion mechanism played an important role in the recent crisis following Lehman Brothers' bankruptcy. Market

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participants stopped lending to one another, as many were perceived to have been exposed to some counterparty risk because of Lehman Brothers' bankruptcy.

This paper establishes a formal model of this phenomenon. In our model, financial institutions finance their long-term investments through short-term liabilities comprising short-term deposits and interbank loans.[1] Thus, these institutions are interconnected through interbank loans, and there is a maturity mismatch between their assets and liabilities, which could potentially lead to bankruptcy of these institutions resulting from a lack of liquidity. We demonstrate that due to short-term depositor uncertainty about the interconnections among these financial institutions, a negative shock to one financial institution can spread to all the other financial institutions in the system, leading to systemic market freezes.

To illustrate this mechanism in the most transparent fashion, we assume that all financial institutions are divided into an equal number of borrowers and lenders. Each borrower is linked to a single lender, and each lender has only one borrower (later we show how this assumption can be modified to be more realistic). Depositors do not have perfect information about the financial institution lending network. Instead, they believe that each lending institution could lend to each of the borrowing institutions in the system with equal probability. This assumption introduces uncertainty about interconnections among financial institutions into the model and plays a key role in our contagion mechanism. In addition, we assume that a negative shock hits the long-term investments of one of the borrowing institutions, which we call the distressed institution. The shock spreads to the rest of the financial institutions in the following way:

First, the financial institution lending to the distressed institution may suffer a loss because its interbank loans may default. Second, depositors of the lending institutions may charge a higher interest rate (we call this case a partial market freeze) or even refuse to roll over their deposits (we call this case a complete market freeze) because of their concern that the financial institution they are lending to may be the one lending to the distressed institution and may thus fail to repay its debts. As a result, the lending institutions may be forced to liquidate their long-term investments and recall their interbank loans. Third, due to the interbank loan recall of the lending institutions, the healthy borrowing institutions not directly affected by the shock could incur a loss. Their depositors may also refuse to roll over their loans, forcing these healthy borrowing institutions to liquidate part or all of their long-term investments.

Our model produces the following major results: First, the severity of a market freeze increases in the magnitude of the negative shock to the distressed institution. As the shock increases, the lending institutions face first no market freeze, then a partial market freeze, and eventually a complete market freeze. In the worst case scenario where the shock is sufficiently large, a system-wide collapse can happen where all the institutions suffer a complete market freeze.

1. Here, we alter the commonly understood definition of "deposits" and use it to refer instead to short-term debts financed through small creditors. Similarly, "interbank loans" is used to refer to financial transactions between any financial institutions, some of which are not necessarily commercial banks.

Second, contagion occurs among financial institutions who need not be interconnected through *actual* financial transactions. Instead, as long as a financial institution is *perceived* by the market to be connected to the distressed institution, it becomes part of the contagion. This perceived connection arises because depositors cannot distinguish between lending institutions and their different exposures to the distressed institution. Thus, our model reveals that short-term depositor uncertainty over network structures can significantly increase the magnitude of contagion compared to a situation without short-term depositor uncertainty.

Third, the size of a large creditor will affect market freezes substantially.[2] In our model, a borrowing institution has two types of creditors: a large creditor (a lending institution) and a continuum of small creditors (depositors). Our model shows that (i) As the size of the large creditor increases, the distressed borrowing institution faces a less severe market freeze, whereas the healthy borrowing institution faces a more severe market freeze. (ii) There is a nonmonotonic relationship between the size of large creditors and the severity of a market freeze faced by the lending institutions.

We find that these results are driven by different incentives of a large creditor when facing different borrowing institutions.[3] For a large creditor lending to the insolvent distressed institution, his interbank loan recall decision is affected by two competing incentives: the incentive to preempt depositors and the incentive to internalize the liquidation cost of the distressed institution. The former incentive induces him to join depositors for a bank run by recalling interbank loans early, whereas the latter incentive induces him to hold interbank loans until the long-term investments of the borrowing institution mature. The large creditor has the latter incentive because he owns a substantial share of the distressed institution's asset. Unlike an individual depositor, his recall decision will affect the liquidation cost of the distressed institution. This liquidation cost is borne by the large creditor when the distressed institution is insolvent. As the size of the large creditor increases, his former incentive becomes weaker, while his latter incentive becomes stronger. In contrast, for a large creditor lending to the solvent healthy borrowing institution, he has no incentive to internalize the borrowing institution's liquidation cost, which is borne by the borrowing institution, instead of by the large creditor himself.

Our model produces the following major policy implications: First, we examine the information policy of a central bank. When the central bank can credibly reveal the financial network structure to the market, we find that contagion can be effectively prevented. In situations in which the central bank does not have perfect information about the financial network structure, we find that more information provided by the central bank about the network structure does not necessarily improve social welfare, potentially making the situation worse.

Second, we examine bailout policies and find that both direct guarantees for the distressed institution and injections of capital to all the lending institutions can alleviate market freezes if the moral hazard problem is ignored.

2. We thank the anonymous referee for encouraging us to examine this issue.

3. We provide a detailed explanation about these incentives when introducing the models.

Third, we examine the lender of last resort (LOLR) function and find that a central bank loan with an interest rate lower than the prevailing market rate available to lending institutions will lower the market rate and make complete market freezes less likely.

One important theoretical contribution of our paper is to introduce a large creditor in the presence of a continuum of small creditors to a Diamond and Dybvig bank run model and study the strategic interactions between the large and small creditors. To our knowledge, our paper is the first to do so. As a result, our model could shed light on the relationship between the size of large creditors and market freezes, and provide theoretical guidance for financial regulators to design a more stable financial system.

Another contribution of our paper is that we complete the financial network literature by introducing incomplete information and strategic interactions among financial institutions. The network literature studies how complex financial networks cause contagion in a financial system through credit chains.[4] This literature usually assumes that agents have perfect information about the financial network structure and that contagion is caused by *actual* connections to the distressed institution.[5] Our paper assumes short-term depositor uncertainty about the financial network structure. As a result, contagion in our model does not rely on *actual* connections to the distressed institution, but *perceived* connections. In the network literature, agents usually follow a mechanical decision rule without any strategic interactions. Our model allows strategic interactions among all the agents in a financial system. As a result, our model examines market dynamics in a financial crisis in a more realistic way.

Contagion in our model does not rely on complex network structures, but is caused by short-term depositor *uncertainty* about network structures. In our model, we adopt a very simple financial network structure in which all banks are paired as borrower and lender. This simple network structure is sufficient to convey the core contagion mechanism in our model.

Our paper is most closely related to Caballero and Simsek (2013) and Pritsker (2013), two studies that examine financial contagion caused by uncertainty in an interconnected financial system. However, both papers use the concept of Knightian uncertainty. In addition, the basic setup of our model is quite different from theirs. For example, our model assumes two types of creditors as mentioned before.

Moreover, our paper contributes to the literature on financial contagion. The existing literature on financial contagion focuses mainly on two contagion mechanisms. The first works through herding behavior caused by information externalities.[6] The

4. There is a large body of literature on this topic that usually involves complex financial networks. The related work includes Allen and Babus (2009), Allen, Babus, and Carletti (2010), Anand, Gai, and Marsili (2012), Espinosa-Vega and Solé (2010), Gai, Haldane, and Kapadia (2011), and Gai and Kapadia (2010) among many others.

5. This is true only for the financial network literature. In the more general literature on financial contagion, incomplete information often plays a role.

6. The related work includes Chen (1999) and King and Wadhwani (1990) among many others.

second works through credit chains. That is, when financial institutions are linked through financial transactions, a failure of one institution can spread to other institutions in the link through balance-sheet effects, leading to a systemic failure.[7]

Finally, our paper also contributes to the literature on market freezes. Explanations for market freezes in the existing literature include adverse selection caused by asymmetric information, Knightian uncertainty, gambling for resurrection, and preemptive runs of short-term creditors due to future rollover risk.[8] In our model, market freezes in the depositor market are caused by asymmetric information: depositors cannot identify the lending institution actually connected to the distressed institution. As a result, they will charge a higher interest rate to all the lending institutions, or even refuse to roll over their deposits. Our model studies freezes in a financial *system* with multiple financial institutions and two markets: the interbank loan market and depositor market. As a result, our model reveals how market freezes spread from one institution to the rest of the institutions in the system and from the depositor market to the interbank loan market.

The rest of the paper is organized as follows. Section 1 describes the environment of the model with imperfect information. Section 2 studies the special case with perfect information. Section 3 characterizes the equilibrium of the model with imperfect information. Section 4 generalizes the model from the two-connection case to the N-connection case. Policy implications are examined in Section 5. Section 6 discusses possible extensions of the paper. Section 7 concludes. Proofs are given in the Appendix.

1. THE ENVIRONMENT

This is a two-period model with three dates denoted by $t = 0,\ 1$, and 2. There are four banks denoted by L_1, L_2, B_1, and B_2, respectively.[9] Banks L_1 and L_2 are the lending banks, and banks B_1 and B_2 are the borrowing banks with an interbank loan market structure given by Figure 1.

1.1 Initial Balance Sheets

The balance sheets of the lending and borrowing banks at date 0 are given by Table 1. Each lending bank has total deposits of $D_0 + x$ and equity of e_0. On the asset side,

7. In addition to the network literature we mentioned above, the related work includes Allen and Gale (2000), Dasgupta (2004), Kiyotaki and Moore (1997, 2002), and Rochet and Tirole (1996) among many others. Besides these two major mechanisms, Kodres and Pritsker (2002) study contagion between countries combining balance-sheet effects from portfolio rebalancing with asymmetric information across market participants.

8. The related work includes Acharya, Gale, and Yorulmazer (2011), Bolton, Santos, and Scheinkman (2011), Brunnermeier and Oehmke (2013), Caballero and Krishnamurthy (2008), Diamond and Rajan (2011), Easley and O'Hara (2010), and He and Xiong (2012) among many others.

9. The word "bank" is used for convenience. It can be interpreted as a nonbank financial institution as well.

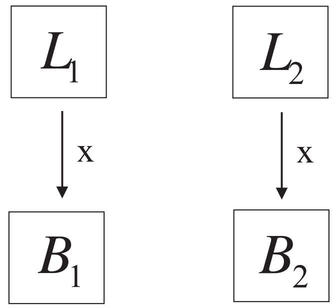


FIG. 1. The Interbank Loan Market Structure. x Is the Interbank Loan Position.

TABLE 1

BALANCE SHEETS AT $t = 0$

Lending banks' balance sheet at $t = 0$		Borrowing banks' balance sheet at $t = 0$	
Assets	Liabilities	Assets	Liabilities
Interbank loan: x	Deposit : $D_0 + x$		Deposit : $D_0 - x$
Long-term project: L	Equity: e_0		Interbank loan: x
		Long-term project: L	Equity: e_0

each lending bank has an interbank loan of x and a long-term project of $L = D_0 + e_0$. Each borrowing bank has total deposits of $D_0 - x > 0$ and equity of e_0. In addition, they borrow an interbank loan of x. Thus, $0 < x < D_0$. It is straightforward to see that the size of the long-term project of each borrowing bank is also $L = D_0 + e_0$.

We assume that the long-term project will mature at date 2 with a net return rate of $R > 0$. If it is liquidated at date 1, for y units of *date 2* output, the liquidation income at date 1 is $\lambda y - 0.5\gamma y^2$, where $0 < \lambda < 1$ and $\gamma > 0$ are constants. Note that $0.5\gamma y^2$ is convex, which captures an increasing marginal liquidation cost and introduces the possibility of nonlinear outcomes. Moreover, note that y denotes *date 2* output. Let l denote the units of long-term project liquidated at *date 1*. Then, $y = (1 + R)l$, and the liquidation income could also be written as $\lambda l(1 + R) - 0.5\gamma[l(1 + R)]^2$. We assume that the marginal liquidation income of $\lambda - \gamma y$ is positive for all the values of y so that a bank will always earn positive income by liquidating an additional unit of the project.

We assume that banks can access only one-period short-term deposits. Thus, they will have to roll over their deposits at $t = 1$ if they invest in long-term projects at $t = 0$. This assumption captures the maturity mismatch between assets and liabilities in a real financial institution. Finally, we assume that the lending banks and depositors are risk neutral and expect at $t = 0$ that their lending will be repaid for sure. Thus, both the interbank loan rate and deposit rate at $t = 0$ equal the riskless rate of zero.

Thus, each borrowing bank has two types of creditors: a large creditor (a lending bank) and a continuum of small creditors (depositors). Moreover, there are two markets in this economy: an interbank loan market and a depositor market. Note that the bank equity market is not included because we assume that e_0 is exogenously given.

One key assumption in our model is that depositors of L_1 and L_2 know L_1 and L_2 as the lending banks and B_1 and B_2 as the borrowing banks. However, they do not know who is lending to whom. As a result, from the perspective of the lending banks' depositors, there are two possible states. In one state, L_1 is lending to B_1 and L_2 is lending to B_2. In the other state, L_1 is lending to B_2 and L_2 is lending to B_1. The depositors believe that each state could happen with equal probability. This assumption introduces short-term depositor uncertainty about the financial network structure to our model.

1.2 The Timing of the Model

At date 1, an unanticipated negative shock hits the long-term project of one of the borrowing banks. Without loss of generality, let B_1 be the one hit by the shock. As a result, its net project return becomes $\hat{R} = R - R_{shock}$. Here, $R_{shock} \in (0, 1 + R)$ measures the magnitude of the negative shock. We assume that the identity of the bank hit by the shock is publicly known.

At date 1, after observing the shock to B_1, depositors and banks make their decisions in the following sequence.

First, the *lending banks' depositors* decide whether to roll over their deposits to the banks, and if they do, what interest rate to charge.

Second, the *lending banks* make their decisions. If their depositors are willing to roll over their deposits, then the two lending banks will decide how many deposits to roll over and how many deposits to repay. Banks may repay part of their deposits by (i) recalling interbank loans and (ii) liquidating the long-term project. We examine the general case where banks can recall part of interbank loans or liquidate part of the long-term project. We also assume that L_1 and L_2 aim at maximizing their net asset value at date 2, even when the net asset value is negative. We believe that this assumption is realistic, because it would be more difficult for a bank manager to find a new job following a more substantial loss in his previous job. Thus, the manager has an incentive to minimize losses when the bank is insolvent.[10] Given this assumption, L_1, who suffers the interbank loan loss, will still have an incentive to roll over its deposits.

Third, the *borrowing banks' depositors* decide whether to withdraw their deposits or not at date 1, after they observe the interbank loan recall decision of the lending banks.

10. The main results of our paper do not depend on this assumption. The alternative assumptions that $\hat{R}$ is uncertain and the lending banks maximize their expected positive equity value will produce similar results but greatly complicate calculations. We thus use the simplified assumption that $\hat{R}$ is certain and the lending banks maximize their net asset value even when it is negative.

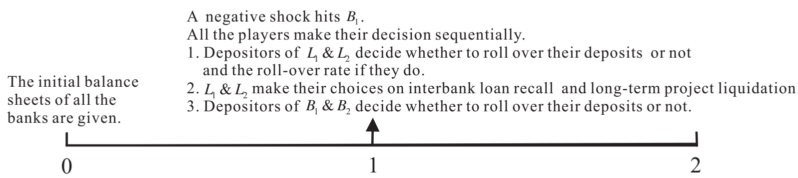


FIG. 2. The Timeline.

Finally, at date 2, banks repay their debts.

We assume that if a bank's total assets are not enough to meet the claims from all the creditors at date 1 or 2, its assets will be shared by all the creditors proportionally to their claims (principal plus interest) in that period. Note that if this situation occurs at date 1, the interbank loans not recalled or deposits not withdrawn at date 1 will receive nothing at date 2, because the bank is liquidated at date 1.

Figure 2 gives the timeline of this model.

Thus, at date 1, there is a sequential game where the lending banks' depositors move first, the lending banks move second, and the borrowing banks' depositors move last. We assume that as long as a no-run equilibrium is feasible, depositors will coordinate toward it. Thus, we rule out an equilibrium in which depositors initiate a bank run at date 1 because of their self-fulfilling beliefs and focus on "essential bank runs" caused by economic fundamentals as in Allen and Gale (1998).

Throughout the paper, we assume that $e_0 < [D_0(1 - R)]/(1 + R)$. Our later analysis shows that this assumption ensures that a lending bank which lends to the distressed bank can become insolvent as long as R_{shock} and x are sufficiently large.

Next, we will examine a special case with perfect information where the identity of L_1 who has lent to B_1 is publicly known. This simpler case will help us better understand an imperfect information case where the identity of L_1 is not publicly known. Later in Section 3, we will return to the imperfect information case.

2. A SPECIAL CASE WITH PERFECT INFORMATION

In the case of perfect information, after the negative shock hits B_1, only L_1 and B_1 are affected. Banks B_2 and L_2 are unaffected. Without any uncertainty, each depositor either withdraws his deposits at date 1 or rolls over his deposits at the riskless rate.

We use backward induction to find the subgame perfect Nash equilibrium as follows. We start with the last mover, B_1's depositors, then move backward to the second mover, L_1, and subsequently the first mover, L_1's depositors.

2.1 Agents' Optimal Choices

Optimal choices for B_1's depositors. We start with the last mover, B_1's depositors, who decide whether to withdraw their deposits or not at date 1, after observing L_1's interbank loan recall decision. Let αx be the amount of interbank loans that L_1 recalls. Thus, α is the proportion of interbank loans that L_1 recalls. Let $NV^{nr}_{B_1,\alpha}$ denote B_1's net asset value at date 2 without experiencing a run, after αx of interbank loans are recalled. Let $V_{B_1,liq}$ denote B_1's asset value when its whole long-term project is liquidated at date 1. Thus,

$$V_{B_1,liq} = \lambda L(1+\hat{R}) - \frac{1}{2}\gamma(L(1+\hat{R}))^2. \tag{1}$$

When $\alpha x \leq V_{B_1,liq}$, we have

$$NV^{nr}_{B_1,\alpha} = (L - l(\alpha))(1+\hat{R}) - [D_0 - x + (1-\alpha)x], \tag{2}$$

where $l(\alpha)$ is determined by

$$\alpha x = \lambda l(1+\hat{R}) - \frac{1}{2}\gamma[l(1+\hat{R})]^2. \tag{3}$$

Note that $l(\alpha) < L$ when $\alpha x < V_{B_1,liq}$, because B_1 has positive assets left for period 2 after L_1 recalls αx of its interbank loans.

When $\alpha x > V_{B_1,liq}$, B_1 cannot fully repay its recalled interbank loans even when it liquidates its entire long-term project. In this case, $NV^{nr}_{B_1,\alpha} = -[(D_0 - x) + (x - V_{B_1,liq})] = -(D_0 - V_{B_1,liq}) < 0$, where $D_0 - x$ is the deposits, and $x - V_{B_1,liq} > 0$ is the unpaid interbank loans. We arrive at the following lemma:

Lemma 1. *When $NV^{nr}_{B_1,\alpha=0} \geq 0$ and $NV^{nr}_{B_1,\alpha=1} \leq 0$, there exists a unique level of $0 \leq \alpha_1 \leq 1$ such that B_1's depositors will withdraw at date 1 if, and only if, $\alpha > \alpha_1$. When $NV^{nr}_{B_1,\alpha=0} < 0$, B_1's depositors will withdraw at date 1 for all the values of $\alpha \in [0, 1]$. When $NV^{nr}_{B_1,\alpha=1} > 0$, B_1's depositors will never withdraw at date 1 for all the values of $\alpha \in [0, 1]$.*

Proof. See the Appendix. □

Optimal choices for L_1. Next, we move backward through the sequence to find out the optimal choice of α for L_1 when it does not experience a bank run. Here, we need not consider the optimal choice for L_1 when it experiences a bank run, because we are confining our attention to "essential bank runs" in which depositors initiate a bank run if, and only if, a no-run equilibrium is infeasible. As a result, we need only to determine whether a no-run equilibrium is feasible. Note that provided that L_1's depositors roll over their deposits, L_1 will never liquidate its long-term project because liquidation is costly. Thus, L_1 needs only to choose the optimal level of α to maximize its net asset value, given the best responses of B_1's depositors. We prove that in equilibrium, L_1 will recall either all or none of its interbank loans, even when it is allowed to recall its interbank loans partially.

We define L_1's interbank loan payoff as its *date 2* value of total proceeds from interbank loans. Thus, if L_1 recalls αx of interbank loans at *date 1*, its payoff from this proportion of interbank loans is $\alpha x(1+r)$, where r is the interest rate charged

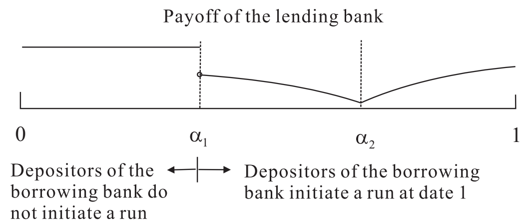


FIG. 3. L_1's Interbank Loan Payoff When $\alpha_1 \in [0, 1]$ and $\alpha_2 > \alpha_1$.

by depositors. Because r is zero in this perfect information case, the payoff equals the sum of cash payments that L_1 receives from B_1 over dates 1 and 2. Let α_2 be the solution to $V_{B_1,liq} = (D_0 - x) + \alpha_2 x = D_0 - (1 - \alpha_2)x$. Recall that $V_{B_1,liq}$ is given by equation (1). $\alpha_2 x$ could be interpreted as a *hypothetical* threshold level of interbank loan recall above which B_1 fails to repay its creditors even when it liquidates its whole long-term project at date 1, conditional on B_1's depositors withdrawing at date 1. When $\alpha > \alpha_2$ and B_1's depositors withdraw at date 1, B_1's liquidation value at date 1 is not enough to repay its liabilities, and thus, this amount will be shared by L_1 and B_1's depositors proportionally to their withdrawal. Note that α_2 is hypothetical and could be negative or greater than 1. When $\alpha_2 < 0$, B_1 has no assets left for date 2 conditional on its depositors withdrawing at date 1, even when L_1 does not recall any interbank loans. On the other hand, when $\alpha_2 > 1$, B_1 has positive assets left for date 2 conditional on its depositors withdrawing at date 1, even when L_1 recalls all the interbank loans.

Bank L_1's interbank loan payoff depends crucially on α_1 and α_2. Note that α_2 can be either lower or higher than α_1. When the liquidation cost is high (or λ is low and γ is high), α_2 will be small. In addition, if $NV^{nr}_{B_1,\alpha=1} \leq 0$ (i.e, $\alpha_1 \in [0, 1]$ or $NV^{nr}_{B_1,\alpha=0} < 0$), we have $\alpha_2 \leq 1$.[11]

We illustrate the intuition behind L_1's recall decision using a special case where $0 \leq \alpha_1 < \alpha_2 \leq 1$. Lemma 2 summarizes L_1's interbank loan payoff in this case.

LEMMA 2. *When* $0 \leq \alpha_1 < \alpha_2 \leq 1$, *$L_1$'s interbank loan payoff is x when* $\alpha \in [0, \alpha_1]$, *is strictly decreasing in α when* $\alpha \in (\alpha_1, \alpha_2)$, *and is strictly increasing in α when* $\alpha \in [\alpha_2, 1]$. *Moreover, L_1's payoff has a downward jump at α_1 and is continuous at* α_2.

PROOF. See the Appendix. □

Figure 3 illustrates the results in Lemma 2. Note that when $\alpha \in [0, \alpha_1]$, a no-run equilibrium is feasible and B_1 is solvent. Thus, L_1's payoff is x. When $\alpha \in (\alpha_1, \alpha_2)$, B_1 is insolvent and its depositors will initiate a run. However, because $\alpha < \alpha_2$, B_1 still has positive assets left for date 2. Because B_1 is insolvent, L_1 will seize all of

11. The proof is provided in the technical appendix (https://jmcb.osu.edu/forthcoming-papers) that is available on the *Journal of Money, Credit and Banking* website.

B_1's remaining assets at date 2. In this case, any interbank loan recall is equivalent to L_1 liquidating its own long-term project at date 1, which is costly. Thus, L_1's payoff is decreasing in α. However, once α reaches α_2, B_1 has no assets left for date 2. In this case, when L_1 increases α, its share in B_1's liquidation value at date 1 also increases. Thus, L_1's payoff is increasing in α. At α_1, there is a downward jump in L_1's payoff. This is because, when L_1 recalls $\alpha x > \alpha_1 x$ of its interbank loans, B_1's depositors switch from a no-run equilibrium to a run equilibrium, inducing more liquidation for B_1 that reduces L_1's payoff.

We assume that when L_1 receives the same payoff from recalling different proportions of interbank loans, it always chooses to recall the minimum proportion of interbank loans. Thus, in our special case, the optimal α is zero. Considering all the possible combinations of α_1 and α_2, we arrive at the following proposition.

PROPOSITION 1. *In a perfect information case, if $NV^{nr}_{B_1,\alpha=0} \geq 0$, L_1 will recall no interbank loans, and B_1's depositors will not withdraw at date 1. Otherwise, B_1's depositors will withdraw at date 1, and L_1's optimal choice is to recall either no loans ($\alpha = 0$) or all the loans ($\alpha = 1$).*

PROOF. See the technical appendix (https://jmcb.osu.edu/forthcoming-papers). □

Note that $NV^{nr}_{B_1,\alpha=0} = L(1 + \hat{R}) - D_0$ according to equation (2). Thus, $NV^{nr}_{B_1,\alpha=0} \geq 0$ could also be written as $L(1 + \hat{R}) \geq D_0$. When this condition holds, B_1 is always solvent without experiencing a run at $\alpha = 0$. Thus, a no-run equilibrium is feasible at $\alpha = 0$ such that L_1's interbank loan payoff is always maximized at $\alpha = 0$.

The above result describes the optimal choice for L_1 that maximizes its interbank loan payoff, which also maximizes L_1's asset value at date 2 conditional on its depositors not withdrawing at date 1, $V^{nr}_{L_1}$. This is because $V^{nr}_{L_1} = L(1 + R) + P_{L_1}$, where P_{L_1} denotes L_1's maximum interbank loan payoff.

Optimal choices for L_1's depositors. Given the maximum asset value of L_1, L_1's depositors decide whether to withdraw at date 1 or not. Following a similar argument to the one for the decisions of B_1's depositors, we arrive at the following proposition:

PROPOSITION 2. *If $V^{nr}_{L_1} \geq D_0 + x$, L_1's depositors will not withdraw at date 1. The optimal interbank loan recall strategy of L_1 and the corresponding best responses of B_1's depositors are characterized in Proposition 1. If $V^{nr}_{L_1} < D_0 + x$, depositors of L_1 and B_1 will withdraw at date 1. Both B_1 and L_1 will be insolvent and thus will be forced to liquidate their entire long-term projects at date 1.*

PROOF. See the Appendix. □

2.2 Equilibrium Outcomes

Equilibrium strategies and R_{shock}. We first examine how agents' optimal choices are affected by R_{shock} at a given level of x. Provided that L_1 will become insolvent if

R_{shock} is sufficiently large,[12] we find that agents' strategies can be described by four threshold levels of R_{shock}: R_1^s, R_2^s, $R_{2,c}^s$, and R_3^s.

COROLLARY 1. *(i) Bank B_1's depositors follow a trigger strategy in which they withdraw at date 1 if, and only if, $R_{shock} > R_1^s$. (ii) Bank L_1 follows a trigger strategy in which it recalls all of its interbank loans if, and only if, $R_{shock} > R_2^s$. (iii) Bank L_1's depositors follow a trigger strategy in which they withdraw at date 1 if, and only if, $R_{shock} > R_3^s$. (iv) Conditional on L_1's depositors not withdrawing at date 1, L_1 follows a trigger strategy in which it optimally chooses to recall all of its interbank loans if, and only if, $R_{shock} > R_{2,c}^s$. (v) We have $R_2^s = \min(R_{2,c}^s, R_3^s)$. (vi) We have $R_1^s \leq R_2^s \leq R_3^s$.*

PROOF. See the technical appendix (https://jmcb.osu.edu/forthcoming-papers). □

The intuition behind Corollary 1 is as follows. When $R_{shock} > R_1^s$, a no-run equilibrium is not feasible and B_1 becomes insolvent. As a result, its depositors will initiate a run. Similarly, L_1 becomes insolvent when $R_{shock} > R_3^s$ and its depositors will initiate a run. Conditional on L_1's depositors not withdrawing at date 1, L_1 will optimally choose to recall its interbank loans if, and only if, $R_{shock} > R_{2,c}^s$. If $R_{2,c}^s < R_3^s$, L_1 will be able to optimally choose to recall its interbank loans when $R_{shock} > R_{2,c}^s$, because it has not experienced a run yet at $R_{2,c}^s$, which is below R_3^s. So, the actual threshold for L_1 to recall its interbank loans, R_2^s, is simply $R_{2,c}^s$. However, if $R_{2,c}^s \geq R_3^s$, once $R_{shock} > R_3^s$, L_1 will experience a run and will be forced to recall its interbank loans. As a result, although L_1 would rather recall its interbank loans when $R_{shock} > R_{2,c}^s$ were its depositors not to withdraw, it will actually be forced to recall its interbank loans once $R_{shock} > R_3^s$. As a result, $R_2^s = R_3^s$. In sum, we have $R_2^s = \min(R_{2,c}^s, R_3^s)$. Hence, $R_2^s \leq R_3^s$. In addition, $R_1^s \leq \min(R_2^s, R_3^s)$ because if B_1 does not experience a run, then a no-run equilibrium must be feasible, in which case L_1 will never experience a run or recall any interbank loans. Consequently, when L_1 recalls its interbank loans or experiences a run, B_1 must have experienced a run.

The large creditor and market freezes. In our model, a borrowing bank's creditors consist of both a large creditor (a lending bank) and a continuum of small creditors (depositors). Recall that with an interbank loan position of x, a large creditor optimally chooses the amount of interbank loans to recall, αx, to maximize its net asset value at date 2. Here, α is the proportion of interbank loans that a large creditor recalls. It is interesting to examine how the relative size of the large creditor in a borrowing bank's total liabilities (x/D_0) will affect the equilibrium. We will examine how x affects R_1^s, R_2^s, R_3^s, and $R_{2,c}^s$. The results are summarized in Corollaries 2 and 3.

COROLLARY 2. *There exists a threshold level of x, $\bar{x}$, such that when $x \leq \bar{x}$, $R_{2,c}^s = R_1^s$, and when $x > \bar{x}$, $R_{2,c}^s$ is strictly increasing in x. In addition, $R_{2,c}^s$ is continuous in x.*

PROOF. See the technical appendix (https://jmcb.osu.edu/forthcoming-papers). □

12. Our assumption of $e_0 < [(1 - R)D_0]/(1 + R)$ ensures that this condition is satisfied when x is sufficiently large.

Corollary 2 implies that L_1 is more reluctant to recall its interbank loans with a larger interbank loan position. This is because L_1's recall decision is affected by two competing incentives: the incentive to preempt B_1's depositors and the incentive to internalize B_1's liquidation cost. The former incentive induces it to recall interbank loans at date 1 ($\alpha = 1$), whereas the latter incentive induces it to hold interbank loans to period 2 ($\alpha = 0$). L_1 has the latter incentive because it owns a substantial share of B_1's assets. As a result, its recall decision will affect B_1's asset liquidation. Meanwhile, because B_1 is insolvent, B_1's liquidation cost will be borne by L_1. As x increases, the former incentive becomes weaker, whereas the latter incentive becomes stronger. To understand this, consider the case where x is so small (or equivalently deposits, $D_0 - x$, are so large) that B_1 has no assets left for period 2 after repaying its depositors at date 1. In this case, L_1 has a strong incentive to preempt depositors, because it will receive nothing without doing so. Meanwhile, it has no incentive to internalize B_1's liquidation cost. This is because B_1 always needs to liquidate its entire long-term project to repay its depositors, regardless of L_1's recall decision. However, when x is large, B_1 will have positive assets left for period 2 after repaying its depositors at date 1. By holding its interbank loans to period 2, L_1 will save B_1's asset from liquidation. Moreover, a higher x will save more liquidation costs for B_1, inducing a stronger incentive for L_1 to internalize B_1's liquidation cost.

Note that unlike the large creditor (L_1), an individual small creditor (depositor) has the sole preemptive incentive and no incentive to internalize B_1's liquidation cost. This is because his size is too small to affect B_1's liquidation cost.

COROLLARY 3. *When $x \leq L(1 + R) - D_0$, L_1 is always solvent and will never experience a run. Thus, R_3^s does not exist. When $x > L(1 + R) - D_0$, R_3^s is continuous in x and there is a unique level or region of x, x^*, below which R_3^s is decreasing in x and above which R_3^s is increasing in x. Moreover, when x is below x^*, $R_{2,c}^s < R_3^s$. While when x is above x^*, $R_{2,c}^s > R_3^s$. When x is at the unique level of x^*, $R_{2,c}^s = R_3^s$.*

PROOF. See the technical appendix (https://jmcb.osu.edu/forthcoming-papers). □

The intuition of Corollary 3 will be discussed in the following numerical example.

2.3 A Numerical Example

We provide a numerical example to illustrate the results in Corollaries 2 and 3. In the numerical examples throughout this paper, we will use the same baseline parameterization in which $e_0 = 0.08$, $L = 1$, $D_0 = L - e_0 = 0.92$, $R = 0.05$, $\lambda = 0.92$, and $\gamma = 0.4$. Note that here, we do not intend to calibrate the economy. Instead, we use the numerical examples to illustrate the qualitative results of our model. We let e_0 be 0.08 to ensure that the capital/assets ratio equals the capital adequacy ratio of 8% required by Basel Accords. We let $\lambda = 0.92$ and $\gamma = 0.4$ such that an entirely liquidated project is worth approximately 70% of its unliquidated value. Note that changes of these parameter values will not affect our qualitative results. Nonetheless, the choice of the values of two key parameters, e_0 and x, does affect the magnitude of contagion greatly. We will find more severe contagion with a lower equity e_0 or

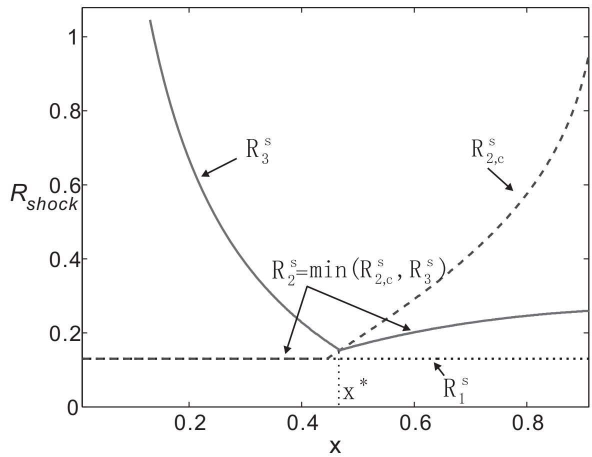


FIG. 4. How R^s_2, $R^s_{2,c}$, and R^s_3 Change in x.

higher interbank loans x. In addition, we let R_{shock} vary from 0 to 1.04 and let x vary from 0.01 to $D_0 - 0.01 = 0.91$.

Figure 4 reveals the results.[13] R^s_3, given by the V-shaped solid line, shows that when $x < x^*$, R^s_3 is decreasing in x. This is because when $x < x^*$, L_1's optimal recall strategy is $\alpha = 1$ at R^s_3.[14] When $\alpha = 1$, L_1's net asset value is decreasing in x, because one unit increase in x has two effects on L_1. First, it increases L_1's assets by $V_{B_1,liq}/D_0 < 1$. Second, it increases L_1's liabilities $(D_0 + x)$ by 1. As a result, L_1's net asset value is decreasing in x. Since L_1's depositors will initiate a run if, and only if, L_1's net asset value is negative, it takes a smaller R_{shock} for L_1's depositors to initiate a run as x increases. To understand the first effect, note that when $\alpha = 1$, L_1 shares B_1's liquidation value, $V_{B_1,liq}$, proportionally with B_1's depositors, and its interbank loan payoff is $(x/D_0)V_{B_1,liq}$. In addition, $V_{B_1,liq}/D_0 < 1$ because for all $R_{shock} > R^s_1$, B_1's total assets at date 2 without any liquidation are lower than its total liabilities, D_0. Thus, its liquidation value, $V_{B_1,liq} < D_0$ as well because of costly liquidation. Thus, we can see that the key reason that L_1's net asset value is decreasing in x here is that a larger x leads to a larger loss in L_1's interbank loan payoff because L_1 is repaid less than one for each unit of its interbank loans.

Figure 4 shows that when $x > x^*$, R^s_3 is increasing in x. This is because in this case, L_1's optimal interbank loan recall strategy is $\alpha = 0$ at R^s_3.[15] When $\alpha = 0$, L_1's

13. In this numerical example, x^* is a unique *level*. In the technical appendix (https://jmcb.osu.edu/forthcoming-papers), we also provide an example in which x^* is a unique *region*. We ignore the latter case because the qualitative results are similar in these two cases.

14. To see why L_1 is more likely to choose $\alpha = 1$ when x is small, review Corollary 2.

15. To see why L_1 is more likely to choose $\alpha = 0$ when x is large, review Corollary 2.

net asset value is increasing in x, because one unit increase in x has two effects on L_1. First, it increases L_1's assets by $1/[\lambda - \gamma l_{B_1}(1 + \hat{R})] > 1$. Second, it increases L_1's liabilities by 1. As a result, L_1's net asset value is increasing in x. Thus, it takes a larger R_{shock} for L_1's depositors to initiate a run as x increases. To understand the first effect, note that when L_1 holds its interbank loans to period 2, one unit increase in x means one unit decrease in deposits withdrawn at date 1, which is given by $D_0 - x$. As a result, $1/[\lambda(1 + \hat{R}) - \gamma l_{B_1}(1 + \hat{R})^2]$ units of B_1's long-term project will be saved from being liquidated to repay one unit of deposits at date 1, which will yield $1/[\lambda - \gamma l_{B_1}(1 + \hat{R})]$ units of proceeds at date 2. Because L_1 will seize all of B_1's assets at date 2 when $\alpha = 0$, it means that L_1's assets will increase by $1/[\lambda - \gamma l_{B_1}(1 + \hat{R})]$. Thus, we can see that the key reason that L_1's net asset value is increasing in x here is that a larger x saves more of B_1's long-term project from costly liquidation.

Figure 4 reveals that $R^s_{2,c}$, given by the dashed line, is nondecreasing in x. In particular, when x is small, it stays at R^s_1. We find that $R^s_2 = \min(R^s_{2,c}, R^s_3)$ is also nondecreasing in x. The R^s_2 curve consists of two parts: When $x < x^*$, it is given by the $R^s_{2,c}$ curve with the dashed line. When $x > x^*$, it is given by the R^s_3 curve with the solid line. Note that at x^*, $R^s_3 = R^s_{2,c}$. This is because R^s_3 is continuous, implying that at x^*, L_1's net asset values at $\alpha = 1$ and $\alpha = 0$ are both zero at R^s_3. It means that at the crossing point, L_1's interbank loan payoffs when $\alpha = 1$ and $\alpha = 0$ are the same, which is exactly the condition that determines $R^s_{2,c}$.

Based on Corollaries 2 and 3, we find that the size of the large creditor, x, affects market freezes as follows. First, a higher x alleviates market freezes for B_1. To see this, note that B_1's interbank loans are less likely to be recalled with a larger x, since R^s_2 is nondecreasing in x. On the other hand, x does not affect the withdrawal decision of B_1's depositors, because R^s_1 is independent of x.

Second, there is a nonmonotonic relationship between x and the severity of market freezes for L_1. When $x < x^*$, R^s_3 is decreasing in x, implying that L_1 is more likely to experience a run with a larger x. However, when $x > x^*$, R^s_3 is increasing in x, implying that L_1 is less likely to experience a run with a larger x.

3. EQUILIBRIUM IN AN IMPERFECT INFORMATION CASE

In this section, we study the imperfect information case where the identity of L_1, who has lent to the distressed bank, is not publicly known. Thus, the two lending banks' depositors will now make the same decisions about deposit withdrawing, because they cannot distinguish between L_1 and L_2. As a result, in contrast to the perfect information case where L_2 and B_2 are unaffected, L_2 and, consequently, B_2 are also affected by the shock to B_1. Thus, our model provides a contagion mechanism in which contagion spreads to L_2 and B_2 because of short-term depositor uncertainty regarding the interconnections among financial institutions.

3.1 Optimal Choices for the Lending Banks

We start with the optimal choices for the lending banks when the market rate charged by their depositors, denoted by $\hat{r}$, is given. Our later analysis reveals that there are two possible cases. In the first case, the lending banks' depositors are willing to roll over their deposits and ask for an interest rate of $\hat{r} \geq 0$. When $\hat{r} > 0$, we call it a partial market freeze. In the second case, there exists no $\hat{r}$ at which the depositors are willing to roll over their deposits, and depositors of L_1 and L_2 will withdraw at date 1. We call this case a complete market freeze.

When $\hat{r} = 0$, both lending banks behave the same as in the perfect information case, conditional on their depositors not withdrawing at date 1. That is, they will optimally choose the proportion of interbank loans to recall, α, as characterized in Section 2.1, and will never liquidate any long-term projects.

When $\hat{r} > 0$, a lending bank will maximize its net asset value at date 2 by optimally choosing the proportion of interbank loans to recall, α, the amount of the long-term project to liquidate, l, and consequently, the optimal amount of deposits to roll over. Note that a lending bank may optimally choose to repay all of its deposits at date 1. Here, we focus on a more interesting case where both the lending banks will roll over a positive amount of their deposits in equilibrium such that both of them participate in the deposits market at date 1. We characterize the optimal decisions of the lending banks in the technical appendix (https://jmcb.osu.edu/forthcoming-papers). In general, their decisions are made similarly to those in the perfect information case, except that now their opportunity cost of not recalling interbank loans or liquidating long-term projects is the positive market rate that they are charged, rather than zero.

3.2 Equilibrium Market Rate

Now we examine how the lending banks' depositors determine the equilibrium market rate they charge. Recall that we assume that they are the first mover and promise to roll over their deposits at the rate of $\hat{r}$. The lending banks are the second mover: at the given $\hat{r}$, they make their optimal decisions. Let V_{L_i} denote L_i's maximum asset value at date 2 under the optimal choices at a given level of $\hat{r}$, where $i = 1$ or 2. Let D_{L_i} denote the optimal amount of deposits that L_i chooses to roll over at a given level of $\hat{r}$. Note that both V and D are functions of $\hat{r}$ and are endogenously chosen by the banks. The conditional probability that the deposits will be rolled over by bank L_i is given by $\pi_{L_i} = D_{L_i}/(D_0 + x)$.

An individual depositor knows that his bank will be good or bad with a 50% probability. In addition, he will take into account the conditional probability of his deposit being rolled over. Given the market rate, a risk-neutral depositor will be willing to roll over his deposit if his expected rate equals the riskless rate. If his expected rate is lower than the riskless rate, he will withdraw at date 1. The equilibrium market rate is determined by:

$$1 = \frac{1}{2}\left[\pi_{good} \min\left(\frac{V_{good}}{D_{good}}, 1 + \hat{r}\right) + (1 - \pi_{good})\right]$$

$$+\frac{1}{2}\left[\pi_{bad}\min\left(\frac{V_{bad}}{D_{bad}},1+\hat{r}\right)+(1-\pi_{bad})\right]. \tag{4}$$

The left-hand side is the return rate if the depositor withdraws at date 1. The right-hand side is the expected return rate from promising to roll over the deposit at $\hat{r}$. With a 50% probability, the depositor's bank is good. In this case, with a probability of $1-\pi_{good}$, the bank will repay its depositor at date 1, and the depositor will receive one unit of payment; with a probability of π_{good}, his deposit is rolled over. In this case, if $1+\hat{r}$ is smaller than V_{good}/D_{good}, he will receive the promised payoff of $1+\hat{r}$ for each unit of his deposits at date 2. Otherwise, all the assets of the bank at date 2 are evenly allocated to the remaining depositors, and the depositors receive the recovery rate of V_{good}/D_{good}. A similar argument is applied to the second term when the bank is bad.

If R_{shock} is so low that depositors know that the bad lending bank can pay the riskless rate for sure (that is, $V_{bad}/D_{bad}\geq 1$), then the good bank should also be able to pay it. In this case, depositors will simply charge the riskless rate of zero, that is, $\hat{r}=0$. This case is identical to the perfect information case where L_1's depositors do not initiate a run and roll over their deposits at a zero interest rate.

For the remaining analysis, we will focus on the more interesting case where R_{shock} is high enough such that $V_{bad}/D_{bad}<1$. Thus, equation (4) can be written as

$$1=\frac{1}{2}\left[\pi_{good}\min\left(\frac{V_{good}}{D_{good}},1+\hat{r}\right)+(1-\pi_{good})\right]$$
$$+\frac{1}{2}\left[\pi_{bad}\frac{V_{bad}}{D_{bad}}+(1-\pi_{bad})\right]. \tag{5}$$

In the special case where the banks choose not to roll over any deposits such that D_{good} or D_{bad} is zero, the corresponding π_{good} or π_{bad} will be zero, and we define the term $\pi(V/D)$ as zero.

The above equation suggests the following steps to find the numerical solutions of the equilibrium market rate. At a given level of $\hat{r}$, let $\Gamma(\hat{r})$ denote the required return rate that must be received from the good bank to satisfy the equilibrium condition:

$$1=\frac{1}{2}\left[\pi_{good}(1+\Gamma(\hat{r}))+(1-\pi_{good})\right]+\frac{1}{2}\left[\pi_{bad}\frac{V_{bad}}{D_{bad}}+(1-\pi_{bad})\right]. \tag{6}$$

If for $1+\hat{r}\leq V_{good}/D_{good}$, we can find a value for $\hat{r}$ such that $\Gamma(\hat{r})=\hat{r}$, then an equilibrium market rate exists. In this case, the required rate is smaller than the maximum payment V_{good}/D_{good} that can be paid by the good bank, so the actual payment is $1+\hat{r}$. Alternatively, if for $1+\hat{r}>V_{good}/D_{good}$, we find $1+\Gamma(\hat{r})=V_{good}/D_{good}$, then an equilibrium market rate also exists. In this case, the required payment $1+\Gamma(\hat{r})$ equals the maximum payment that can be paid by the good bank, and it gives the depositors an expected net return rate of zero. We call the former case the type I equilibrium, and the latter case the type II equilibrium. If for all the values of $\hat{r}$, the required rate, $1 + \Gamma(\hat{r})$, is higher than $1 + \hat{r}$ or V_{good}/D_{good}, then an equilibrium market rate will not exist, and a complete market freeze will occur. Note that in a type I equilibrium, excess returns reside for the good lending bank because the maximum return rate that the good lending bank can afford, V_{good}/D_{good}, is above $1 + \hat{r}^*$. These excess returns will go to equity holders.

In the above analysis, we use subscripts "good" and "bad" to denote the good and bad lending banks. This is because the lending banks' depositors cannot distinguish between L_1 and L_2 but only know the optimal actions that will be taken by each type of bank. In the true state, the actual good and bad banks are L_2 and L_1, respectively. In the following analysis, we use the actual values of π_{L_2}, V_{L_2}, and D_{L_2} to replace π_{good}, V_{good}, and D_{good}, respectively. The similar argument is applied to π_{L_1}, V_{L_1}, and D_{L_1}.

3.3 *The Equilibrium*

Based on the above analysis, the equilibrium in the imperfect information case can be characterized as follows.

PROPOSITION 3. *An equilibrium market rate, $\hat{r}^*$, exists when (i) given $\hat{r}^*$, the two lending banks maximize their net asset value at date 2 by optimally choosing the amount of long-term projects to liquidate and the proportion of interbank loans to recall, and (ii) given the expected optimal choices of the lending banks, $\hat{r}^*$ satisfies equation (5). If the required rate $\Gamma(\hat{r})$ is always higher than $\hat{r}$ or $V_{L_2}/D_{L_2} - 1$, then an equilibrium market rate does not exist. In this case, the lending banks' depositors will not roll over their deposits.*

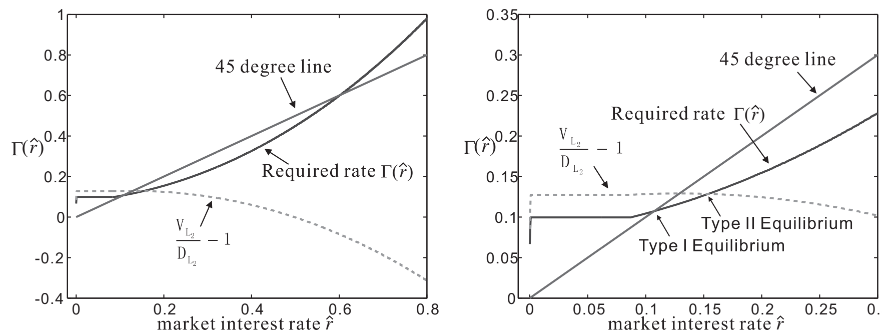
3.4 *Equilibrium Outcomes*

Here, we provide numerical examples to illustrate equilibrium outcomes. We first present a benchmark case where $R_{shock} = 0.32$ and $x = 0.6$ to illustrate how the equilibrium market rate charged by the lending banks' depositors, $\hat{r}^*$, is determined. Figure 5 shows the result. Here, panel (b) is a closeup of part of panel (a) for a clearer presentation.

We find both type I and type II equilibria in this example. In the type I equilibrium, $\Gamma(\hat{r})$ crosses the 45º line below $(V_{L_2}/D_{L_2}) - 1$. More specifically, $\Gamma(\hat{r}) = \hat{r} = 0.1080 < (V_{L_2}/D_{L_2}) - 1 = 0.1287$. Note that in this equilibrium, $\hat{r}^* < (V_{L_2}/D_{L_2}) - 1$, implying that excess returns will go to L_2's equity holders. In the type II equilibrium, $\Gamma(\hat{r})$ crosses $(V_{L_2}/D_{L_2}) - 1$ below the 45ºline. More specifically, $\Gamma(\hat{r}) = (V_{L_2}/D_{L_2}) - 1 = 0.1285 < \hat{r} = 0.1544$.[16]

This example reveals that there may exist multiple equilibria in which the lending banks' depositors charge different market rates. We focus on the equilibrium with

16. $\Gamma(\hat{r})$ is determined by the optimal choices of L_1 and L_2 on interbank loan recall and long-term project liquidation that consequently determine variables such as D_{L_1}, V_{L_1}, D_{L_2}, V_{L_2}, π_{L_1}, π_{L_2}, V_{L_1}/D_{L_1}, and V_{L_2}/D_{L_2}. The details about the determination of $\Gamma(\hat{r})$ are provided in the technical appendix (https://jmcb.osu.edu/forthcoming-papers).



(a) $\Gamma(\hat{r})$ (b) $\Gamma(\hat{r})$ when $\hat{r} \leq 0.3$

FIG. 5. Determination of $\hat{r}^*$ at $R_{shock} = 0.32$.

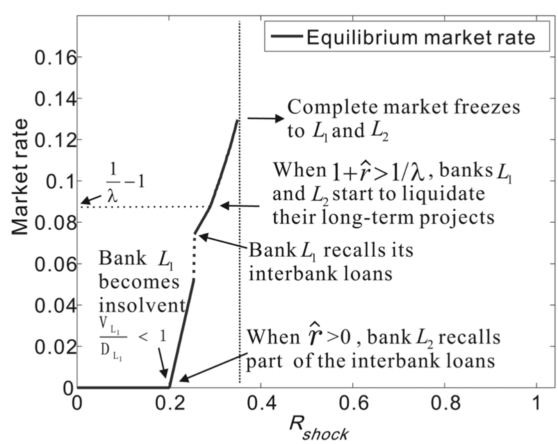


FIG. 6. How the Equilibrium Market Rate Varies in R_{shock}.

the smallest market rate, which implies that contagion is, at the very least, as severe as our results can reveal.[17] Following this rule, Figure 6 shows how $\hat{r}^*$ changes when R_{shock} varies from 0 to 1.04 at $x = 0.6$. It reveals that when $R_{shock} \leq 0.2018$, $\hat{r}^* = 0$ because L_1 is solvent. When $R_{shock} > 0.2018$, $\hat{r}^*$ becomes positive, because L_1 becomes insolvent, implying that L_1's depositors will receive a return rate lower than one. $\hat{r}^*$ is increasing when $R_{shock} > 0.2018$, because L_1 incurs a higher loss from its interbank loans with a higher R_{shock}, inducing a lower *ex post* return rate for its depositors. When $R_{shock} > 0.3494$, there exists no equilibrium market rate, and a complete market freeze occurs.

17. Note that a higher market rate induces lending banks to recall more interbank loans, leading to more liquidation and a higher default risk for borrowing banks.

Note that there is an upward jump of $\hat{r}^*$ at $R_{shock} = 0.2538$, because L_1 starts to recall all of its interbank loans and uses the proceeds to repay its depositors. This will lead to a lower return rate for the remaining depositors of L_1, inducing a higher market rate.

Moreover, note that once $\hat{r}^*$ becomes positive, L_2 starts to recall $\alpha_1^{L_2} = 0.8344$ of its interbank loans to repay its depositors, which is socially inefficient. This is because as long as $\alpha \leq \alpha_1^{L_2}$, B_2 is solvent, and the private cost for L_2 to recall interbank loans is zero. On the other hand, its opportunity cost of not recalling is the positive market rate. As a result, L_2 will recall at least $\alpha_1^{L_2}x$ of interbank loans when the market rate is positive. However, the social cost for L_2 to recall interbank loans, that is, the liquidation cost incurred by B_2 due to L_2's recall, is positive. L_2 will not internalize this liquidation cost borne by B_2. Thus, at the social level, there is too much liquidation. This result suggests one source of liquidity shortage and inefficiency during financial crises: when facing higher funding costs during a crisis, creditors start to recall loans from solvent borrowers, regardless of the high social costs of doing so.

Figure 6 reveals the following major results: (i) the severity of market freezes increases in the magnitude of the negative shock to the distressed bank, B_1. The market experiences first no market freeze, then a partial market freeze, and eventually a complete market freeze as R_{shock} increases. (ii) Contagion spreads as follows: First, L_1 may suffer a loss because of its interbank loans to B_1. Second, the lending banks' depositors may charge a positive market rate or even refuse to roll over their deposits because they suspect that their lending bank may lend to B_1 and incur a loss. Third, the healthy borrowing bank, B_2, may suffer a loss because its lending bank L_2 may recall its interbank loans when facing a higher rollover rate or even experiencing a run by its depositors. In the worst case scenario, a systemic bank run occurs in which all the banks experience a run by their depositors and are forced to liquidate their entire long-term projects.

3.5 The Large Creditor and Market Freezes

Now we examine how the size of the large creditor, x, affects market freezes in the imperfect information case. Note that B_1's depositors follow the same withdrawal strategy as in the perfect information case. That is, they withdraw at date 1 if, and only if, $R_{shock} > R_1^s$, where R_1^s is independent of x. Moreover, the threshold level of R_{shock} for L_1's depositors to initiate a run in the perfect information case, R_3^s, now becomes the threshold level above which the market rate is positive and below which the market rate is zero. This is because when $R_{shock} > R_3^s$, L_1 becomes insolvent and fails to repay its depositors fully. As a result, the depositors will charge a positive market rate to compensate for their expected loss from lending to L_1.

Due to the complex nature of the problem, we cannot provide an analytical analysis on other agents' behavior. However, our numerical analysis reveals that the intuition that we gained in the perfect information case can be applied to this case. Here,

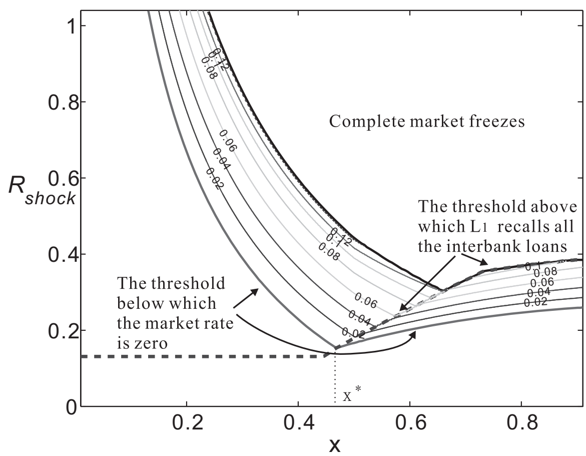


FIG. 7. L_1's Interbank Loan Recall Decision and Equilibrium Market Rates.

we provide a numerical example with the same parameter values as in the perfect information case. The results are illustrated in Figure 7.

Figure 7 shows how L_1's interbank loan recall strategy changes in x. In our example, L_1 follows a trigger strategy in which it recalls all of its interbank loans when R_{shock} is above a threshold level and recalls no interbank loans when R_{shock} is below the threshold level. This threshold curve is given by the dashed line. Note that when $x < x^*$ where x^* is defined in the perfect information case, this threshold curve is identical to the R_2^s (or equivalently $R_{2,c}^s$) curve in the perfect information case because L_1 makes the optimal interbank loan decision at a zero market rate. Thus, it faces the same decision problem as in the perfect information case. When $x > x^*$, first, we find that the threshold curve is above the R_2^s (or equivalently R_3^s) curve in the perfect information case. Thus, L_1 is less likely to recall its interbank loans with imperfect information than with perfect information. This is because in the perfect information case, L_1 is forced to recall interbank loans as long as R_{shock} exceeds R_3^s due to the bank run it experiences. But in the imperfect information case, when R_{shock} just exceeds R_3^s, L_1 faces a positive market rate, instead of a bank run. Thus, L_1 may recall its interbank loans at a threshold level of R_{shock} higher than R_3^s. Thus, compared to the perfect information case, B_1 faces a milder market freeze. Second, we find that this threshold curve is increasing in x. Thus, L_1 is less likely to recall its interbank loans with a larger x, implying that a larger x alleviates a market freeze for B_1. Similar to the perfect information case, the intuition here is that L_1 has a stronger incentive to internalize B_1's liquidation cost with a larger x.

Figure 7 also reveals how the equilibrium market rate that L_1 and L_2 are charged varies in x and R_{shock}. Each of the V-shaped contour lines indicates all the combinations of R_{shock} and x that yield the same level of equilibrium market rates. For example, on the bottom contour line, each point represents a combination of R_{shock}

and x that yields an equilibrium market rate of zero. Strictly speaking, this contour line is the upper bound below which all the combinations of R_{shock} and x yield a zero equilibrium market rate. We just explained that it is identical to the R_3^s curve in the perfect information case. Similarly, the contour line right above the bottom one marked with 0.02 gives all the combinations of R_{shock} and x that yield an equilibrium market rate of 0.02. We find that above the threshold curve for L_1 to recall its interbank loans (when x is low), the market rate increases in x at a given R_{shock}. Similar to the perfect information case, this is because L_1's net asset value, given by $\alpha = 1$, is decreasing in x. As a result, L_1's deposit return rate is lower with a higher x. Thus, depositors will require a higher market rate to compensate for their loss from lending to L_1 as x increases. On the other hand, below the threshold curve for L_1 to recall interbank loans (when x is high), the market rate decreases in x at a given R_{shock}. Similar to the perfect information case, this is because L_1's net asset value, given by $\alpha = 0$, is increasing in x, and we have the opposite result. Thus, similar to the perfect information case, there is a nonmonotonic relationship between x and the severity of market freezes for L_1 and L_2.

Moreover, Figure 7 reveals that at a given level of x, the market rate is zero below R_3^s, and increases in R_{shock} when $R_{shock} > R_3^s$ until a complete market freeze occurs when R_{shock} is sufficiently high. Thus, we find that L_1 is less likely to experience a run in the imperfect information case than in the perfect information case. The intuition is straightforward: with imperfect information, L_1 is thought to be a good bank with a probability of 50%. As a result, it faces a milder market freeze.

However, imperfect information leads to market freezes for L_2 and B_2, who are unaffected under perfect information. We just explained the market freeze L_2 faces in the depositor market. B_2 is affected by L_2's interbank loan recall decision as follows. When the market rate is zero, L_2 recalls no interbank loans. When the market rate is positive, define a hypothetical interbank loan recall level, x_{run}, above which B_2's depositors start to initiate a run. L_2 will recall at least $\min(x, x_{run})$ of interbank loans. This is because once the market rate is positive, L_2 will optimally choose to recall its interbank loans to repay its depositors as long as B_2 is solvent, which is the case when the interbank loan recall is no greater than x_{run}. When the market rate does not exist and a complete market freeze occurs, L_2 could be forced to recall all of its interbank loans, which occurs in our numerical example. In addition, B_2's depositors will initiate a bank run only when L_2's interbank loan recall exceeds x_{run}.

Note that B_2 can face a more severe market freeze with a higher x. In particular, when the market rate is positive and $x < x_{run}$, L_2 will simply recall its whole interbank bank loans of x. Thus, a higher x leads to a more severe market freeze for B_2. Recall that for B_1, a higher x leads to a milder market freeze because L_1 becomes more reluctant to recall interbank loans. The key reason for this difference is that in the case of B_1 who is insolvent, a higher x induces a stronger incentive for L_1 to delay interbank loan recall to avoid a higher liquidation cost of B_1, which will eventually be borne by L_1. However, in the case of B_2, as long as it is solvent, it will be able to fully repay its interbank loans, and the liquidation cost will be covered by B_2's equity. As a result, L_2 will not bear B_2's liquidation cost. Thus, L_2 is more willing to recall

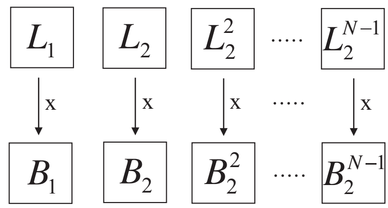


FIG. 8. The Interbank Loan Market Structure with N Pairs of Banks. x Is the Interbank Loan Position.

interbank loans than L_1. This result is interesting because one common perception is that a financial system mainly relying on small creditors tends to be more stable than one mainly relying on large creditors. Our model shows that funding from a large creditor can be either more or less stable than funding from small creditors, depending on the degree to which the large creditor internalizes the liquidation cost of the borrowing bank caused by its interbank loan recall.

4. THE GENERAL CASE OF N CONNECTIONS

4.1 Equilibrium

Now we extend the model to a more general case where B_1 is perceived to be connected to N lending banks. Assume that there are N pairs of banks. The bank that lends to B_1 is still called L_1. The remaining $N-1$ lending banks are called L_2-type banks and the remaining $N-1$ borrowing banks are called B_2-type banks. Figure 8 gives the interbank loan market structure.

The sole difference between the 2-pair and N-pair cases is that the expected return to all the lending banks' depositors who roll over their deposits now changes to:

$$1 = \frac{N-1}{N}\left[\pi_{L_2}\min\left(\frac{V_{L_2}}{D_{L_2}}, 1+\hat{r}\right) + (1-\pi_{L_2})\right] + \frac{1}{N}\left[\pi_{L_1}\min\left(\frac{V_{L_1}}{D_{L_1}}, 1+\hat{r}\right) + (1-\pi_{L_1})\right]. \tag{7}$$

That is, all the lending banks' depositors believe, with a probability of $(N-1)/N$, that their bank is an L_2-type bank, and, with a probability of $1/N$, that their bank is an L_1-type bank. This difference will lead to different equilibrium market rates for the lending banks. However, all the previous analysis about the optimal decisions for the lending and borrowing banks in the 2-pair case at a given $\hat{r}$ can be applied to the N-pair case.

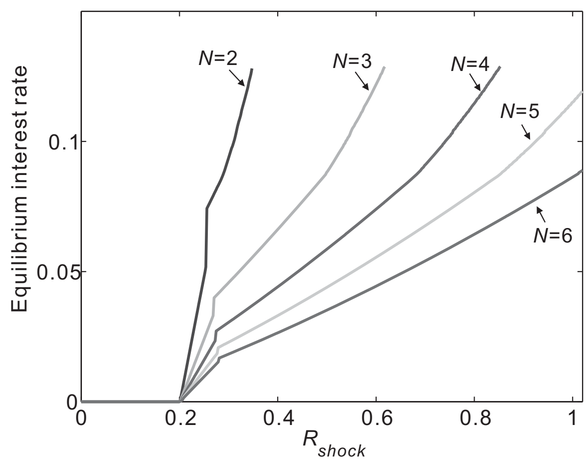


Fig. 9. How $\hat{r}^*$ Changes in R_{shock} When B_1 Is Connected to N Lending Banks.

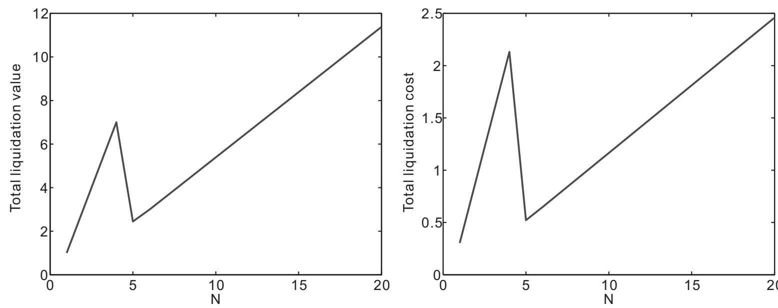
4.2 *A Numerical Example*

Here, we give numerical examples to illustrate the model, using the same baseline parameterization. In addition, we assume that R_{shock} varies from 0 to 1.04 and $x = 0.6$.

Figure 9 shows how $\hat{r}^*$ changes in R_{shock} for $N = 2$, 3, 4, 5, and 6. First, we find that $\hat{r}^*$ becomes positive for all the Ns when R_{shock} exceeds 0.2018. This is because when R_{shock} exceeds this level, L_1 becomes insolvent. Second, at a given level of R_{shock}, the equilibrium market rate is lower with a higher N. This is because $1/N$, the *ex ante* probability that a lending bank is L_1 is decreasing in N; thus, the required rate $\Gamma(\hat{r})$ is also lower for the same R_{shock} when N becomes larger, leading to a lower equilibrium interest rate. Due to the same reason, with a higher N, the threshold level of R_{shock} above which a complete freeze occurs also becomes higher. In our example, a complete market freeze for the lending banks occurs when $N = 2$, 3, and 4, after R_{shock} reaches 0.3494, 0.6198, and 0.8549, respectively. When $N > 4$, no complete market freeze occurs at any level of R_{shock}. Thus, we find that the market freeze is less severe with a higher N.

We also find that at a given level of R_{shock}, the total liquidation value may have a nonmonotonic relationship with N. Here, we define the total liquidation value as the total amount of long-term projects liquidated by all the banks except B_1.[18] This nonmonotonic relationship is caused by the trade-off in long-term project liquidation when N increases. On the one hand, as we showed in Figure 9, a higher N induces a lower $\hat{r}^*$, necessitating less liquidation of an individual bank's long-term project. On the other hand, the total liquidation value of long-term projects may increase because more banks are involved in the contagion.

18. We exclude B_1 to focus on the social cost caused purely by contagion.



(a) Total liquidation value (b) Total liquidation cost

FIG. 10. Total Liquidation Value and the Associated Total Liquidation Cost with Different Ns at $R_{shock} = 0.88$. (B_1 Is Excluded. $N = 1$ Represents the Perfect Information Case.)

Figure 10 gives a numerical example at $R_{shock} = 0.88$ to illustrate the nonmonotonic relationship between N and the total liquidation value and the associated total liquidation cost. Here, we define the total liquidation cost as the sum of the liquidation cost of all the banks except B_1. The liquidation cost of an individual bank is given by $(1 + R)l - \{\lambda l(1 + R) - 0.5\gamma[l(1 + R)]^2\}$, where $(1 + R)l$ is the potential date 2 output that the bank's liquidated long-term project, l, would yield if it were not liquidated, and $\{\lambda l(1 + R) - 0.5\gamma[l(1 + R)]^2\}$ is the actual proceeds that the bank's liquidated long-term project yields. In this example, a complete market freeze occurs at $N = 2$, 3, and 4 with imperfect information and also occurs with perfect information. However, no complete market freeze occurs when $N > 4$ with imperfect information. As a result, the total liquidation value of all the banks except B_1 is $2N - 1$ when $N \leq 4$, which is strictly increasing in N. The total liquidation cost is given by $0.3045 \times (2N - 1)$, because the cost of liquidating a healthy bank's entire project is 0.3045. However, at $N = 5$, there is a downward jump of the total liquidation value because, in this case, L_1- and L_2-type banks do not experience a run. All the lending banks will liquidate a small amount of their long-term projects (0.01) because $\hat{r}^* > 1/\lambda - 1 = 0.087$. In addition, L_2-type banks will recall $\alpha_1^{L_2} x = 0.8344x = 0.5006$ of their interbank loans, forcing all the B_2-type banks to liquidate $l = 0.6006$ of their long-term projects that yield a proceed of 0.5006. The liquidation cost incurred by each B_2-type bank is thus $l(1 + R) - 0.5006 = 0.13$. Similarly, the liquidation cost of each lending bank is 0.00086. Thus, the total liquidation value is $0.6006 \times 4 + 0.01 \times 5 = 2.4524$, and the associated total liquidation cost is $0.13 \times 4 + 0.00086 \times 5 = 0.5243$. When $N \geq 6$, only B_2-type banks will liquidate their long-term projects to meet the interbank loan recall from L_2-type banks. As a result, the total liquidation value is given by $0.6006 \times (N - 1)$, and the associated total liquidation cost is given by $0.13 \times (N - 1)$. Note that as long as $\hat{r}^* > 0$, L_2-type banks will recall $\alpha_1^{L_2} x = 0.8344x$ of their interbank loans, inducing the liquidation of B_2-type banks' long-term projects. In addition, the total liquidation cost under imperfect information could be much higher than that under perfect information ($N = 1$), implying that contagion due to short-term depositor uncertainty about financial structures could greatly magnify the total loss across the whole financial system.

5. POLICY IMPLICATIONS

In this section, we explore the policy implications of our model. We examine three major policies: the information policy, the bailout policy, and the LOLR policy.

5.1 Information Policy

Contagion in our model is caused by short-term depositor uncertainty about the identity of the exposed lending bank. If this information can be revealed to the market, market freezes will not spread to the healthy lending and borrowing banks. Thus, our model demonstrates that it is critical for a central bank to keep track of the financial network structure and reveal it to the market in a credible way all the time. This function of the central bank is particularly important during a financial crisis, because solvent banks will have difficulty in credibly identifying themselves as solvent to the market.

It is interesting to examine a case where a central bank can help reduce the uncertainty about the identity of the distressed lending bank but does not have perfect information. In this case, less uncertainty does not necessarily improve social welfare. We demonstrated previously that the total liquidation cost is not a monotonic function of N, the number of possible distressed lending banks. A central bank can reduce N by identifying some healthy banks among them. However, as long as it cannot identify all the healthy banks, more information may lead to more long-term project liquidation and lower social welfare. For example, suppose that the central bank reduces the number of the possible distressed banks from N to $N - 1$ by identifying one of them as healthy. Because the probability for each lending bank to be distressed now increases, a market freeze may be more severe. It is possible that no complete market freeze occurs with N possible distressed banks, but a complete market freeze occurs with $N - 1$ possible distressed banks. In this case, the social cost could be much higher with less uncertainty.

5.2 Bailout Policy

Guaranteeing B_1's debt. In our model, contagion originates from B_1. A straightforward way to prevent contagion is to remove the originator; that is, to bail out B_1 by using taxpayers' money to pay the losses of B_1's creditors. As a result, the bank lending to B_1 is saved and the contagion is stopped. In our simple model, this method is effective and easy to implement. Note that our model and, consequently, our policy analysis does not taken into account moral hazard. In reality, this concern may make the government reluctant to use this policy.[19]

Injecting capital into the lending banks. The government can alleviate market freezes by buying preferred shares or stock issued by the lending banks. Suppose that the preferred stock injected into each bank is sufficient for L_1 to fully repay its depositors at date 2 such that lending banks' depositors will roll over their deposits at a zero market rate. Note that although the government buys preferred shares in the N lending banks, only the preferred shares in L_1 will suffer a loss. Thus, when N increases, the initial funds that the government needs to inject into the banks will increase, but the actual loss that the government incurs will remain constant, which is limited to the loss to L_1 that equals the loss caused by its interbank loans, deducted by the loss absorbed by L_1's equity. Thus, as N increases, the actual cost of saving the banks does not increase, whereas the cost of not saving the banks (i.e., the cost of liquidating healthy banks' long-term projects) could increase, creating a stronger incentive for the government to inject capital into all the lending banks. Of course, here, we ignore moral hazard and the controversy associated with the nationalization of the banking industry.[20]

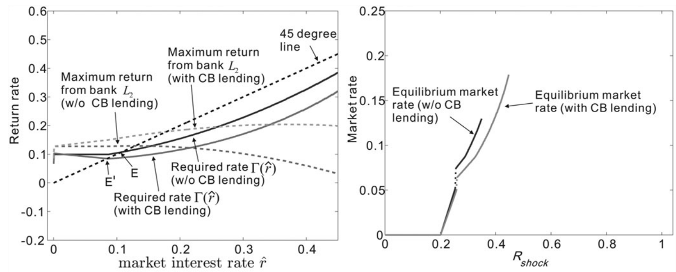
The advantage of this policy, compared to directly bailing out bank B_1, is that the government will use less taxpayers' money to bail out the financial system. The government need not pay all the creditors of B_1, and the loss to L_1 is absorbed first by L_1's equity. In this sense, the moral hazard problem is less severe under this policy than under the policy of direct bailout.

5.3 LOLR Policy

LOLR policy discussions often cite the classic book by Bagehot (1873) in which he summarized the LOLR policy as a central bank lending freely to solvent institutions against good collateral at a higher interest rate. According to Freixas, Parigi, and Rochet (2004), the classic Bagehot rules can be criticized for two reasons: (i) it is difficult for a central bank to distinguish between illiquidity and insolvency as an LOLR (see, e.g., Goodhart 1999), and (ii) as argued by Goodfriend and King (1988), with a well-functioning interbank loan market, the open market operation of central banks is enough to maintain an efficient market, rendering the LOLR policy unnecessary. As Freixas, Parigi, and Rochet observed, although the classic Bagehot rules might be viewed as obsolete, a number of emergency liquidity assistance measures were taken in the past financial crises, but without a consistent framework for doing so.

19. In the recent subprime mortgage crisis, the U.S. government refused to bail out Lehman Brothers due to the concern of moral hazard. This led to severe market freezes in the financial system following the Lehman Brothers' bankruptcy.

20. In reality, the severity of moral hazard associated with this policy will be affected by factors such as the government's final loss from bank recapitalization and the duration that the government holds banks' shares. In the recent subprime mortgage crisis, the U.S. government held the shares of U.S. banks only for a short period, and incurred little loss from bank recapitalization.



(a) Equilibrium at $R_{shock} = 0.32$ with and without central bank lending.

(b) Equilibrium $\hat{r}$ with and without central bank lending.

FIG. 11. The Effect of Central Bank Lending on the Market Equilibrium Rate When $\bar{L}_{CB} = 0.25$ and $r_{CB} = 0$.

Our model can be extended to one where a central bank lends to all the lending banks. Suppose that the central bank has no better information than market participants. Then, the optimal LOLR policy suggested by our model is simply to lend freely at the riskless interest rate. This policy can effectively stop market freezes, minimize long-term project liquidation, and therefore achieve maximum social welfare.

In reality, a central bank tends to lend up to a limit. We make a simple extension of our basic model with $N = 2$ to examine this case. More specifically, we assume that (i) the central bank provides loans up to $\bar{L}_{CB}$ to each of the lending banks, at a fixed interest rate $r_{CB} = 0$, (ii) $\bar{L}_{CB}$ is small so that the lending banks will still need to borrow from the market, and (iii) the lending banks will choose to borrow from the central bank if, and only if, the market rate $\hat{r} > r_{CB}$. Assumption (ii) ensures that lending banks still need to borrow from the market in addition to central bank loans so that we can analyze how central bank lending will affect the market rate. We find that an LOLR policy of limited lending at a rate lower than the prevailing market rate will generally alleviate market freezes and improve social welfare.

Below, we provide a numerical example with the same baseline parameterization as before to illustrate the effects. In addition, we assume that $\bar{L}_{CB} = 0.25$ and $x = 0.6$. Figure 11(a) compares the results with and without central bank lending at $R_{shock} = 0.32$. It reveals that with central bank lending, the curve of the required rate, $\Gamma(\hat{r})$, shifts downward and crosses the 45° line at a lower equilibrium rate. The equilibrium rate is 0.1264 without central bank lending and is 0.1 with central bank lending. In addition, the constraint imposed by the maximum return from the good bank L_2 is now shifted up. Note that this curve is independent of the level of R_{shock}, because the action of the good bank L_2 depends only on $\hat{r}$. This implies that the required rate, $\Gamma(\hat{r})$, is more likely to be below this curve, and a complete market freeze is less likely to happen.

Figure 11(b) shows the results when R_{shock} varies from 0 to 1.04. For values of R_{shock} at which the equilibrium rate is positive without central bank lending, the equilibrium rate is now lower with central bank lending. For some values of R_{shock} at which a complete market freeze occurs without central bank lending, a partial market freeze now occurs with central bank lending.

Because the LOLR policy reduces long-term project liquidation and the associated output loss, it improves social welfare on the aggregate level. But not every group of agents is benefited equally. L_1's depositors are better off due to a lower loss from lending to L_1. However, L_2's depositors may be worse off by earning a lower market rate if the LOLR policy alleviates a partial market freeze. In this case, L_2's shareholders are better off because of a lower funding cost. The borrowing banks' depositors are better off because a less severe market freeze lowers the borrowing bank's liquidation.

6. INTERPRETATIONS AND EXTENSIONS OF THE MODEL

In our model, we assume that there are equal numbers of paired borrowers and lenders. This is an extreme assumption to make the game symmetrical and relatively easy to solve. But we can reinterpret the model to allow for more realistic network credit exposures.

First, assume that the model describes a distressed bank and a group of identical lending banks that are aggregated into one representative creditor bank. Then, we could treat the remainder of the lending banks as institutions that are known to have been exposed to the distressed bank in the past; but depositors are uncertain about their current exposures to the distressed bank. In this interpretation, the other lending banks have no exposure to the distressed bank, but could suffer a complete or partial freeze because of depositor uncertainty.

Second, we can add a further group of institutions that are widely known to depositors as having no exposure to the distressed bank. This group is immune to a freeze.

Third, one can think about our model as a domestic banking system embedded in an international banking system. It is easy to reinterpret our model in a situation where a domestic banking system and domestic short-term money markets are potentially exposed to international credit risks through domestic bank exposures to international credit risks. Although only a subset of domestic banks may be vulnerable, domestic money markets may fear that there are undisclosed exposures.

A critical aspect of our model is that short-term depositors do not have accurate information about the network of bank exposures. We could create a richer theory if we allow short-term depositors to acquire additional information by observing some signals. For instance, in the current model, we implicitly assume that borrowing from the central bank does not change depositors' beliefs. A possible extension is to allow banks' borrowing activities to reveal information about their asset quality. As a result, banks may use central bank loans as a signaling tool, and it would be interesting to find out what the optimal central bank loan policy would be after this informational effect is taken into account. For example, central banks may have an incentive to hide the identity of the borrowing banks, if depositors believe that all the banks borrowing from the central bank are insolvent. In this case, borrowing from the central bank will greatly increase a bank's cost of borrowing from the market, resulting in a lower than socially optimal level of borrowing from the central bank. In the recent subprime mortgage crisis, the Bank of England indeed maintained a policy of never revealing the identity of the banks that took loans from it for this reason.

7. CONCLUSIONS

This paper studies contagion and systemic market freezes caused by short-term creditor uncertainty regarding interconnections in the financial system. Our model reveals that because of short-term creditor uncertainty regarding interconnections, all the financial institutions *perceived* by the market to be connected to the distressed institution can be involved in the contagion, even when they have no *actual* connection to it. Thus, our model shows that the magnitude of contagion could be greatly magnified because of short-term creditor uncertainty about interbank exposures in the financial system. In addition, we examine the role of a large and small creditors in a financial crisis. In general, we find a nonmonotonic relationship between the size of the large creditor and the severity of market freezes that lending institutions face.

Policy implications are also explored in our paper. Using our model, we find that it is crucial for a central bank to keep and disclose accurate information about the financial network structure constantly and consistently to prevent contagion in a financial crisis. Moreover, when in reality, the central bank does not have perfect information, our model shows that better information provided by the central bank to market participants to refine their beliefs about interbank exposures would not necessarily improve social welfare. This is because better information reduces the number of lending institutions possibly exposed to the distressed institution, but increases the probability for each of them to be the one lending to the distressed institution. As a result, they will face a more severe market freeze. If we ignore the moral hazard implications of recapitalization, our model demonstrates that (i) bailout policies guaranteeing the debts of the financial institution hit by the shock, and (ii) capital injections to all the lending financial institutions are two policies that can halt contagion. The LOLR policy can also effectively alleviate market freezes by lowering market funding rates and reducing the incidence of complete market freezes.

APPENDIX A

A.1 Proof of Lemma 1

In the case of $NV^{nr}_{B_1,\alpha=0} \geq 0$ and $NV^{nr}_{B_1,\alpha=1} \leq 0$, we first prove that when $\alpha x \leq V_{B_1,liq}$, $NV^{nr}_{B_1,\alpha}$ is strictly decreasing in α. Note that in this case, $\partial NV^{nr}_{B_1,\alpha}/\partial\alpha = -(1 +$

$\hat{R})(\partial l/\partial\alpha) + x$. Using equation (3), we get $\partial l/\partial\alpha = x/[\lambda(1+\hat{R}) - \gamma l(1+\hat{R})^2]$. Thus, we have

$$\frac{\partial NV^{nr}_{B_1,\alpha}}{\partial\alpha} = -\frac{1}{\lambda - \gamma l(1+\hat{R})}x + x < 0, \tag{A1}$$

since $0 < \lambda - \gamma l(1+\hat{R}) < 1$ by assumption. When $\alpha x > V_{B_1,liq}$, $NV^{nr}_{B_1,\alpha} = -(D_0 - V_{B_1,liq})$ is a negative constant. Note that $NV^{nr}_{B_1,\alpha}$ is continuous in α, because at $\alpha = V_{B_1,liq}/x$, $l(\alpha) = L$ and $NV^{nr}_{B_1,\alpha} = -(D_0 - \alpha x) = -(D_0 - V_{B_1,liq})$.

When $NV^{nr}_{B_1,\alpha=0} \geq 0$ and $NV^{nr}_{B_1,\alpha=1} \leq 0$, because $NV^{nr}_{B_1,\alpha}$ is strictly decreasing in α when $\alpha \leq V_{B_1,liq}/x$, and remains a negative constant when $\alpha > V_{B_1,liq}/x$, there exists a unique level of $0 \leq \alpha_1 \leq 1$ such that $NV^{nr}_{B_1,\alpha_1} = 0$. We will prove that B_1's depositors withdraw at date 1 if, and only if, $\alpha > \alpha_1$ in this case as follows.

First, we prove that B_1's depositors will never withdraw at date 1 when $\alpha x \leq \alpha_1 x$, or $\alpha \leq \alpha_1$. In this case, B_1's net asset value is given by $(L - l(\alpha))(1+\hat{R}) - (D_0 - \alpha x) \geq (L - l(\alpha_1))(1+\hat{R}) - (D_0 - \alpha_1 x) = 0$ because it is strictly decreasing in α. Thus, B_1 is solvent without experiencing a run and is able to fully repay its deposits. Hence, as long as all the other depositors do not withdraw at date 1, an individual depositor has no incentive to deviate. Thus, we prove that a no-run equilibrium is feasible. Because we assume that whenever a no-run equilibrium is feasible, depositors will coordinate toward it, we prove that B_1's depositors will not withdraw at date 1 when $\alpha \leq \alpha_1$.

Second, we prove that B_1 is insolvent when $\alpha > \alpha_1$, conditional on its depositors not withdrawing at date 1. When $\alpha > \alpha_1$, there are two possible situations. First, $\alpha x \leq V_{B_1,liq}$ and B_1's net asset value is $(L - l(\alpha))(1+\hat{R}) - (D_0 - \alpha x) < (L - l(\alpha_1))(1+\hat{R}) - (D_0 - \alpha_1 x) = 0$ because it is strictly decreasing in α. Second, $\alpha x > V_{B_1,liq}$. We already explained that B_1's net asset value is negative in this case. Thus, in both situations, B_1 is insolvent, conditional on its depositors not withdrawing at date 1.

Third, we prove that a no-run equilibrium is infeasible when $\alpha > \alpha_1$. Suppose that all the other depositors do not withdraw at date 1, there are two possible situations. First, B_1 has positive assets left at date 2. In this case, an individual depositor not deviating will receive less than one unit of the good, because B_1 is insolvent. On the other hand, a depositor deviating to withdrawing at date 1 will receive one unit of the good. Thus, a depositor is better off by deviating. Second, B_1 has no assets left at date 2. In this case, a depositor not deviating will receive nothing. On the other hand, a depositor deviating to withdrawing at date 1 will receive a positive payment. Thus, in both situations, a depositor is better off by deviating. Thus, we prove that a no-run equilibrium is not feasible when $\alpha > \alpha_1$.

Fourth, we prove that a bank run equilibrium is actually a Nash equilibrium when $\alpha > \alpha_1$. Suppose that all the other depositors withdraw at date 1. There are two possible situations. First, B_1 has no assets left at date 2. In this case, B_1's total assets, which are strictly positive, will be proportionally allocated to depositors at date 1. Thus, an individual depositor receives a positive payment by withdrawing at date 1 and nothing by deviating to withdrawing at date 2. Second, B_1 has positive assets left at date 2, implying that B_1 must have fully repaid its depositors at date 1. Thus, an individual depositor not deviating receives one unit of the good. But he will receive less than one unit of the good by deviating to withdrawing at date 2 because B_1 is insolvent.[21] In both situations, the depositor has no incentive to deviate. Thus, we prove that a bank run equilibrium is a Nash equilibrium when $\alpha > \alpha_1$. Thus, we prove that B_1's depositors always withdraw at date 1 when $\alpha > \alpha_1$.

In the case of $NV^{nr}_{B_1,\alpha=0} < 0$, B_1 is insolvent without experiencing a run for all $\alpha \in [0, 1]$. As a result, B_1's depositors will always withdraw at date 1 for all $\alpha \in [0, 1]$.

In the case of $NV^{nr}_{B_1,\alpha=1} > 0$, B_1 is solvent without experiencing a run for all $\alpha \in [0, 1]$. As a result, B_1's depositors will never withdraw at date 1 for all $\alpha \in [0, 1]$. □

A.2 Proof of Lemma 2

Let Π denote L_1's total interbank loan payoff in terms of date 2 value. First, when $\alpha \in [0, \alpha_1]$, B_1 has enough assets at date 2 to repay all the creditors. Thus, we have

$$\Pi = \alpha x + (1 - \alpha)x = x. \tag{A2}$$

Second, when $\alpha \in (\alpha_1, \alpha_2]$, we have

$$\Pi = \alpha x + (L - l)(1 + \hat{R}), \tag{A3}$$

$$\alpha x + D_0 - x = \lambda l(1 + \hat{R}) - \frac{1}{2}\gamma[l(1 + \hat{R})]^2. \tag{A4}$$

Using equation (A4), we have

$$\frac{\partial l}{\partial \alpha} = \frac{x}{(1 + \hat{R})(\lambda - \gamma(1 + \hat{R})l)}. \tag{A5}$$

Thus, we have

$$\frac{\partial \Pi}{\partial \alpha} = x - (1 + \hat{R})\frac{\partial l}{\partial \alpha} = x - \frac{x}{\lambda - \gamma(1 + \hat{R})l} < 0, \tag{A6}$$

because by assumption $0 < \lambda - \gamma(1 + \hat{R})l < 1$.

Third, when $\alpha \in [\alpha_2, 1]$, we have

$$\Pi = \frac{\alpha x}{\alpha x + (D_0 - x)} V_{B_1,liq} = \frac{\alpha x}{\alpha x + (D_0 - x)}(\lambda L(1 + \hat{R}) - \frac{1}{2}\gamma[L(1 + \hat{R})]^2). \tag{A7}$$

21. B_1's net asset value is always higher in a no-run equilibrium than in a bank run equilibrium because a bank run leads to costly liquidation. When $\alpha > \alpha_1$, B_1 is insolvent without a bank run. Thus, it must be insolvent with a bank run.

It is straightforward to see that Π is strictly increasing in α. Note that there is a downward jump in Π at α_1, because $(L - l)(1 + \hat{R}) < (1 - \alpha)x$. This jump is caused by B_1's long-term project liquidation due to its depositors' withdrawal at date 1.

In addition, Π is continuous at α_2, that is, $\Pi(\alpha \in (\alpha_1, \alpha_2], \alpha = \alpha_2) = \Pi(\alpha \in [\alpha_2, 1], \alpha = \alpha_2)$. Note that by definition, $\alpha_2 x + (D_0 - x) = V_{B_1,liq}$. As a result, $\Pi(\alpha \in (\alpha_1, \alpha_2], \alpha = \alpha_2) = \Pi(\alpha \in [\alpha_2, 1], \alpha = \alpha_2) = \alpha_2 x$. □

A.3 Proof of Proposition 2

This argument is similar to Lemma 1. When $V_{L_1}^{nr} \geq D_0 + x$, a no-run equilibrium is feasible. Provided that other depositors do not initiate a run, an individual depositor will not initiate a run. When $V_{L_1}^{nr} < D_0 + x$, a no-run equilibrium is not feasible. Provided that other depositors do not withdraw at date 1, an individual depositor will always be better off by withdrawing at date 1, because he will receive one unit of the good by withdrawing at date 1 and will receive less than one unit of the good by withdrawing at date 2. Moreover, a bank run equilibrium is actually a Nash equilibrium: Provided that other depositors initiate a run, an individual depositor will get nothing if he withdraws at date 2. This is because $V_{L_1,liq} < V_{L_1}^{nr} < D_0 + x$ due to costly liquidation caused by the bank run. On the other hand, withdrawing at date 1 will yield a positive payoff of $V_{L_1,liq}/(D_0 + x)$ for L_1's depositors. So, withdrawing at date 1 is a Nash equilibrium.

We prove that when L_1 experiences a bank run, B_1 must have experienced a run as follows. If B_1's depositors do not initiate a run, L_1's depositors will never initiate a run. This is because when B_1 does not experience a run, B_1 must have been solvent at date 2, implying that L_1's interbank loans will be fully repaid at date 2. As a result, L_1 will not incur any losses and be solvent. Thus, a no-run equilibrium is feasible for L_1. Thus, we can infer that when L_1 experiences a run, B_1 must have experienced a run. □

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