7 Risk Neutral Valuation

Risk neutral valuation (RNV) is a very elegant method for pricing risky securities. It was elaborated for the pricing of options and other derivatives, but it can be applied to many other questions such as determining the profitability of a risky investment or assessing the value of complex securities. The reasons for the popularity of RNV are its simplicity and, more importantly, the fact that it allows determination of risk premiums (how much financial markets require to bear different risks) without any knowledge of economic fundamentals. But this is too good to be true. The method relies on two assumptions that are completely unrealistic: all risks are traded on active markets (market completeness) and trade is efficient (no frictions nor transaction costs).

This chapter explores the magic of perfect markets and shows that these two assumptions imply that the RNV method is indeed valid. However, it also shows the mirages of the perfect markets world: putting too much faith into these assumptions leads to some crude fallacies: "maturity transformation is not risky," "leverage does not matter," and "risk management is useless!"

7.1 THE EXPECTED PRESENT VALUE CRITERION

The simplest tool used in finance to evaluate an investment is to compute the expectation of the net present value of future cash flows generated by this investment—for example, a project that needs an investment of $I = 100$ and provides expected cash flows of 60 in each of the next 2 years. Its expected (net) present value is:

$$EPV(r) = \frac{60}{1+r} + \frac{60}{(1+r)^2} - 100.$$

When the interest rate, r, used to discount future cash flows is 5%, we obtain a positive value:

$$EPV(5\%) = 11.56.$$

This suggests that the investment should be undertaken. However this computation does not take into account the risk of future cash flows, because a random cash flow $\tilde{x}$ is valued identically to a certain cash flow with the same mean $\mathcal{E}(\tilde{x})$.

In practice, investors usually require a higher rate of return on risky investments. For example, stocks, which are more volatile than bonds, typically have higher returns (on average, over a long period). The difference is called a risk premium. Similarly, corporate bonds, which are subject to default risk, have higher nominal returns than government bonds, which are normally immune to default.[1] The difference in returns is called the corporate spread: it is usually bigger than what a simple estimation of the probability of default would imply.

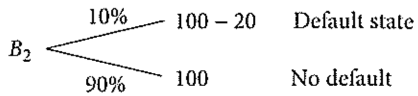

Figure 7.1

Numerical Example:

Consider two bonds that promise to repay the same amount (100) in a 1 year time. Bond 1 is a government bond with yield $r = 5\%$: This means that the price of this bond is:

$$B_1 = \frac{100}{1.05} \simeq 95.2.$$

By contrast, bond 2 is a corporate bond issued by some firm X, characterized by an estimated probability of default $PD = 10\%$ and a loss given default $LGD = 20\%$.

Bond 2 has a higher yield $R = 10\%$ and thus a lower price $B_2 = \frac{100}{1.1} \simeq 90.9$.

Bond 2 being risky, its liquidation value can be modeled as a Bernoulli variable as shown in Figure 7.1.

The expected return of bond 2 is thus

$$\mu = \frac{(0.1)(80) + (0.9)(100)}{90.9} - 1 \simeq 7.8\%.$$

This is higher than the expected return on the risk-free bond, which is only 5%. If the corporate bond had an expected return of 5%, its price would be given by the expected present value of its future repayments:

$$EPV = \frac{(0.1)(80) + (0.9)(100)}{1.05} \simeq 93.3.$$

The price $B_2 = 90.9$ of the bond is smaller than this EPV because most investors are risk-averse.[2] *The difference between the expected return of the risky bond B_2 and that of the risk-free bond B_1 is called a* ***risk premium****.*

Assessing risk premiums is in principle a delicate exercise, requiring good forecasts of future returns on the risky asset, as well as on the future economic situation. However, when sufficiently many financial assets are traded actively, their market prices reveal implicitly the risk premiums required by investors for different types of risks. This is the magic of perfect markets, which we now explain.

7.2 THE MAGIC OF PERFECT MARKETS

Suppose that on the top of the two bonds considered above, a new financial instrument is traded, namely a credit default swap.

This CDS is a contract that guarantees the payment of a fixed sum, say \$1, if firm X defaults. To estimate the reasonable premium P that the protection seller can charge to the CDS buyer is in principle a complex exercise, requiring analysis of the financial situation of firm X and forecast of the future situation of the overall economy.

BOX 7.1 ■ Credit Default Swaps

A credit default swap (CDS) is a contract between two parties. The **protection buyer** pays a periodic premium to the **protection seller** who, in exchange for this premium, commits to pay a fixed sum if a credit instrument (a bond or a loan) goes into default. The mechanism is very similar to insurance, but there are two important differences:

- The protection buyer does not necessarily own the credit instrument that is insured by the CDS contract. CDS contracts may therefore be used for **speculating** as well as **hedging**.
- The protection seller is not necessarily a regulated entity.

The CDS market has grown tremendously since the beginning of the twenty-first century. By the end of 2007, the world CDS market had a (notional)[3] value of more than \$45 trillion! However, the subprime crisis has precipitated a dramatic compression of this market. Regulators are also deeply concerned with the opacity of this market, which made exposures of large financial institutions difficult to monitor. For example, the U.S. insurance giant AIG lost more than \$100 billion on CDS contracts in 2008 (*see* Box 1.1).

The magic of the complete markets world is that the CDS premium P can in fact be computed without any further knowledge just by using the **no arbitrage principle**, which is simply a sophisticated version of the law of one price. Indeed, if one constitutes a portfolio comprising one corporate bond B_2 (of nominal 100 and loss given default of 20) and 20 *CDS* contracts, the total payoff of this portfolio is 100, independently of whether firm X defaults.

States Values	Corporate Bond (B_2)	20 *CDS* contracts	Total portfolio
Default of firm X	80	20	100
No default	100	0	100

This portfolio is exactly equivalent to a risk-free bond. Therefore, the price of the portfolio must equal the price of the risk-free bond, (otherwise an arbitrage is possible):

$$B_2 + 20P = B_1.$$

Using the numerical values $B_1 = 95.2$ and $B_2 = 90.9$, we obtain the unique value of the *CDS* premium P that is compatible with the no arbitrage principle:

$$P = \frac{95.2 - 90.9}{20} = 21.5\%.$$

If we compare P with the actuarially fair premium

$$P_0 = \frac{PD}{1+r},$$

we see that the probability of default that would be necessary to justify such a high level of the *CDS* premium is

$$\widehat{PD} = (1+r)P = (1.05) \times (0.215) \simeq 22.6\%.$$

This is more than the double of the "true" probability of default—that is, the one that is estimated on historical data

$$PD = 10\%.$$

$\widehat{PD}$ is called the risk neutral (*RN*) probability of default.[4] This name comes from the fact that when the expected present value of the corporate bond B_2 (this is also true for any security) is computed under this *RN* probability (we call it the risk neutral value [*RNV*]), it matches the price of B_2 observed in the market

$$RNV = \frac{1}{1+r}\left[\widehat{PD} \times 80 + (1-\widehat{PD}) \cdot 100\right],$$
$$= \frac{1}{1.05}\left[(0.226) \times 80 + (0.774) \cdot 100\right] = 90.9.$$

Thus, $RNV = B_2$.

The spread between the risk-adjusted probability of default $\widehat{PD}$ and the historical probability of default *PD* reflects the market's risk aversion and investors' expectations about the future state of the economy. As a result, it may vary a lot over time.

We can now summarize our findings.

The modern tool of risk neutral valuation is thus a rigorous way to take risk into account while maintaining the simplicity of expected present value computations. The idea is to modify probabilities in such a way that they incorporate risk premia. The modified probabilities, called risk neutral probabilities, can be computed from market prices. Arbitrage pricing methods (initially used to price options but now applied to all sorts of financial computations) show that in the absence of arbitrage opportunities, it is always possible to find such risk neutral probabilities. Moreover, if financial markets are complete, there is a unique way to compute them.

7.3 COMPLETE MARKETS AND ABSENCE OF ARBITRAGE OPPORTUNITIES

To explain the principle of absence of arbitrage opportunities, we need to introduce some notation. Consider a two-period model of a financial market.

At date 0, N financial assets are traded at prices $p_1, \ldots, p_N$.

Uncertainty is modeled by an ordered tree that has one node at date 0 (today) and S nodes at date 1 (tomorrow), corresponding to the different possibilities (called the states of the world) that can occur at that date (*see* Figure 7.2):

At date 1, asset n pays off a value v_{sn} that depends on the state of the world, s, at that date. The matrix $V = (v_{sn})_{\substack{s \le S \\ n < N}}$ has S rows and N columns, where S is

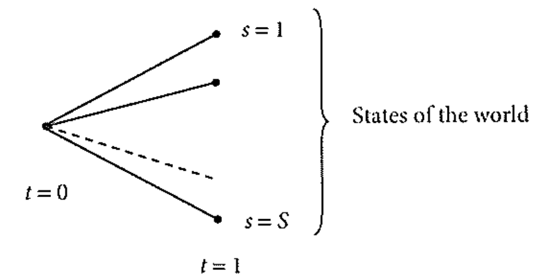

Figure 7.2 Modeling uncertainty by a tree.

the number of states of the world at date 1. A portfolio θ is any vector $(\theta_1, \ldots, \theta_N)$ in $\mathbb{R}^N$, θ_n representing the number of assets n in the portfolio. Note that θ_n can be negative, which means that shortsales are possible. Each portfolio $\theta \in \mathbb{R}^N$ costs $p \cdot \theta = p_1\theta_1 + \cdots + p_N\theta_N$ at date 0 and delivers payoff $m_s = \sum_n v_{sn}\theta_n$ at date 1 if state s prevails. The payoff vector, m, is thus equal to $V\theta$.

Definition 7.1. *The market is arbitrage-free* $\Leftrightarrow \forall \theta \in \mathbb{R}^N,\ V\theta \in \mathbb{R}^S_+ \Rightarrow p \cdot \theta \geq 0$.

An arbitrage would correspond to a portfolio θ that has a negative cost ($p \cdot \theta < 0$) at date 0 and a non-negative payoff vector ($m = V\theta \geq 0$) at date 1.

In the absence of frictions or transaction costs, the market will be (close to) arbitrage-free, as any arbitrage opportunity will be immediately exploited.

Note that in an arbitrage-free market, two portfolios θ_1 and θ_2 that generate the same payoff vector at date 1 (and therefore are such that $V\theta_1 = V\theta_2$) must necessarily have the same price at date 0 (and therefore are such that $p \cdot \theta_1 = p \cdot \theta_2$). This is the **replicating portfolio principle.** We now introduce a second important notion.

Definition 7.2. *The market is complete* $\Leftrightarrow \forall m \in \mathbb{R}^S,\ \exists \theta \in \mathbb{R}^N,\ V\theta = m$.

Thus, a market is complete if and only if the rank of matrix V is S. This means that the space spanned by payoff vectors $V \cdot \theta$ (for all possible portfolios θ) has full dimension S. In other words, any vector, m, in $\mathbb{R}^S$ can be generated as the payoff vector of some portfolio θ in $\mathbb{R}^M$.

Proposition 7.1 (Arbitrage Pricing). *If the market is arbitrage-free and complete, there exists a unique vector* π *in* $\mathbb{R}^S_+$ *(called the vector of state prices) such that for any asset, n:*

$$p_n = \sum_s \pi_s v_{sn}.$$

Thus, the price of any asset is equal to the sum of what it pays in all states of the world, weighted by the vector of state prices.

The proof of Proposition 7.1 is easy: If markets are complete (rank $V = S$), it must be that $N \geq S$ and that a non-singular $S \times S$ matrix of "fundamental" assets can be extracted from V. All other assets can be replicated by portfolios of

BOX 7.2 ■ The Principle of Risk Neutral Valuation

When financial markets are perfect (i.e., arbitrage-free and complete), all financial assets can be priced by computing the expectation of the present value (PV) of their future cash flows under a fictitious probability distribution that neutralizes risk premia.This probability distribution is called the **risk neutral probability**, and the expectation is called the RNV of cash flows. It has been used successfully for option pricing (Black Scholes) and bond pricing (Merton). A particularly simple example is the binomial model of Cox, Ross, and Rubinstein.

these S fundamental assets. Moreover, for each s there is a unique portfolio of fundamental assets that gives a payoff of 1 in state s and 0 is all other states. Since the market is arbitrage-free, the price π_s of this portfolio must be non-negative. This ends the proof of Proposition 7.1.

Proposition 7.1 shows how to compute the price of any security in this model. Consider, in particular, the price of a riskless bond of nominal 1. Because the payoff of this bond is by definition 1 in all states of the world ($v_{sn} \equiv 1$) the price of this bond (given by Proposition 7.1) must be equal to

$$\frac{1}{1+r} = \sum_s \pi_s.$$

Now, we can define a probability distribution $Q = (q_1, \dots, q_s)$ by setting (for all $s = 1, \dots, S$): $q_s = (1+r)\pi_s$.

This allows us to define the related notion of RNV:

Proposition 7.2 (Risk neutral valuation). *If the market is arbitrage-free and complete, there exists a unique probability distribution Q on* $\{1, \dots, S\}$ *(i.e., a vector* $[(q_1, \dots, q_S)]$ *in* $\mathbb{R}^S_+$ *with* $\sum_s q_s = 1$*) such that, for any asset n:*

$$P_n = \frac{1}{1+r} \sum_s q_s v_{sn}.$$

Proposition 7.2 is an immediate consequence of Proposition 7.1 obtained by replacing π_s by $\frac{q_s}{1+r}$.

7.4 A BINOMIAL EXAMPLE

As a second example of the RNV method, consider the binomial model of Cox, Ross, and Rubinstein (1979). This model can be seen as the elementary version of the Black Scholes model in the sense that it gives the economic intuition behind option pricing methods without any need to use the complex techniques of stochastic calculus. This economic intuition can be stated literarily as follows: in the absence of frictions on financial markets, the risk premiums that financial markets assign to different contingencies can be deduced from the observation of the prices of sufficiently many securities.

The Cox-Ross-Rubinstein model allows the simplest possible formulation of this argument. In this model, there are only two dates ($t = 0, 1$) and two contingencies (states of the world) at date 1, say a boom (state up) and a recession (state down). As soon as there are two different securities that are traded, the prices of these securities determine the risk premium required by investors on any other security and thus the "fair" price of any other security. Suppose, for example, that the two traded securities are a riskless bond with (gross) return $1+r$ and a stock with (gross) return u in state up and d in state down (with $d < u$). If $u < 1+r$ or $d > 1+r$, one asset dominates the other. For both assets to be traded, it is necessary that $d \leq 1+r \leq u$. If p denotes the probability of state up (and $1-p$ the probability of state down), the expected return on the stock is higher than the riskless return when:

$$pu + (1-p)d > 1+r.$$

The risk premium on the stock equals, by definition, the difference between the expected return on the stock and the riskless return $(1+r)$:

$$\pi = pu + (1-p)d - (1+r).$$

However, because $d \leq 1+r \leq u$ (otherwise, one of the two assets is dominated by the other) there always exists a (unique) $q \in [0,1]$ such that

$$qu + (1-q)d = 1+r. \tag{7.1}$$

Consider the probability distribution Q that assigns probability q to state "up" (boom) and probability $1-q$ to state "down" (recession). This probability distribution Q is called the risk neutral probability distribution because it neutralizes risk premia. Under Q, the expected (net) return on all assets is equal to the risk-free return r.

Numerical Example: $u = 1.2$, $r = 5\%$, $d = 1$, $p = 1/2$;

$$\pi = pu + (1-p)d - (1+r) = \frac{1}{2}(1.2) + \frac{1}{2}(1) - 1.05$$
$$= 5\%.$$

The risk neutral probability of state up—that is, q—is the unique solution of

$$q \times (1.2) + (1-q) \times (1) = 1.05 \quad \Rightarrow \quad q = 1/4.$$

The risk neutral probability distribution $Q = (1/4, 3/4)$ *can be viewed as more "pessimistic" that the "true" or "historical" probability distribution* $P = (1/2, 1/2)$ *because the probability of the "favorable" event u is reduced from* $1/2$ *to* $1/4$.

What is remarkable is that this risk neutral probability always exists (unless there are arbitrage opportunities). It is unique as soon as markets are complete.[5] Once Q is determined (in our binomial example it suffices to solve (7.1), which gives $q = \frac{1+r-d}{u-d}$ and $1-q = \frac{u-(1+r)}{u-d}$), it is easy to compute the "fair" prices of all assets that pay future cash flows contingent on the stock's returns.

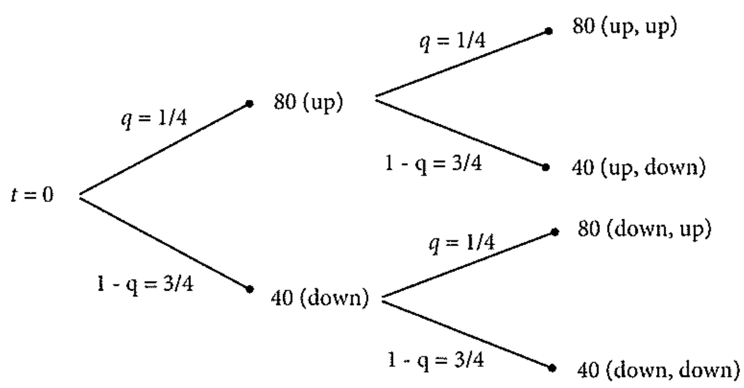

Figure 7.3

Let us come back to our initial example of an investment of $I = 100$ that provides expected cash flows of 60 in each of the next 2 years. If the interest rate is 5% and the cash flows are not risky, we saw that the investment should be undertaken, because its NPV is positive:

$$NPV = \frac{60}{1.05} + \frac{60}{(1.05)^2} - 100 \sim 11.56.$$

If cash flows are risky, the computation has to be modified. Suppose, for example, that cash flows $\tilde{C}$ follow a Bernoulli process, as in the Cox-Ross-Rubinstein model illustrated in Figure 7.3:

Expected cash flows per period are

$$\frac{1}{2}(80) + \frac{1}{2}(40) = 60.$$

However, the risk neutral expectation of cash flows is lower:

$$\mathcal{E}(\tilde{C}) = \frac{1}{4}(80) + \frac{3}{4}(40) = 50.$$

In fact, the risk neutral expected present value becomes negative:

$$RNV = \frac{50}{1.05} + \frac{50}{(1.05)^2} - 100 < 0.$$

Therefore, once risk premia are taken into account, the risky investment should **not** be undertaken! This reverses the previous conclusion, which was based on the EPV method, which neglects risk premia.

7.5 THE MIRAGES OF THE PERFECT MARKETS WORLD

A well-known anecdote[6] illustrates perfectly the dangers of the perfect markets hypothesis: A finance student and her professor find a $100 bill lying on the

ground. The student wants to pick it up, but the finance professor tells her: "Don't bother: if it were a real $100 bill, it would not be here."

To some extent, the illusion that financial markets are close to be perfect (i.e., arbitrage-free, complete, and frictionless) has been responsible for the generalized excess in risk taking by financial institutions that was one of the main characteristics of the subprime crisis. This section develops three examples of fallacies generated by the perfect market assumption: maturity transformation is not risky (7.5.1), leverage does not matter (7.5.2) and risk management is useless (7.5.3).

7.5.1 Fallacy 1: Maturity Transformation is not Risky

One of the major economic functions of banks is to collect deposits from the public and transform then into loans to firms and households. The maturity of the loans is typically larger than a year, whereas that of deposits can be very short (often 1 week or even 1 day). This economy function of banks is called maturity transformation. It exposes banks to the risk of a run by depositors: if, for some reason or another, deposits are not renewed, the bank may be forced to liquidate its long-term assets at a loss, which may even provoke its default.

However, if markets were perfect and banks' assets were reasonably safe, then this situation would never occur, because the liquidation of these assets (whenever needed) would never entail a loss with respect to their fundamental value. Bank defaults would still be possible, but they would be provoked by solvency, rather than liquidity, problems.

In the real world, markets are not perfect, and banks can be exposed to liquidity problems as illustrated below.

Consider a bank that has the following business model (very much in the spirit of Northern Rock): borrow short term at the risk-free interest rate r, and invest long term (say at maturity T) in a (reasonably) safe asset that returns R per period. The (initial) balance sheet of the bank is

Assets (A)	Equity (E) Deposits (D)

Thus, all the bank's debt is in the form of deposits. The bank is subject by regulation to a capital requirement:

$$E \geq kA.$$

After T periods, assets are liquidated for a value $A(1+R)^T$. Deposits only last one period, so that the bank has to borrow

$$D_t = D(1+r)^t$$

at date t to repay the amount $D_{t-1} = D(1+r)^{t-1}$ borrowed at date $t-1$, plus the interests. If markets are perfect, then shareholders are sure to obtain at date

T the difference between the asset value $A(1+R)^T$ and the final debt obligation $D(1+r)^T$. The present value of the gain for shareholders is

$$\pi = \left[A\left(\frac{1+R}{1+r}\right)^T - D\right] - E.$$

Using the fact that $D+E=A$, and assuming that $R>r$, we obtain a positive return on equity

$$ROE = \frac{\pi}{E} = \frac{A}{E}\left[\left(\frac{1+R}{1+r}\right)^T - 1\right],$$

which illustrates well the classical sources of profit for shareholders: leverage $\frac{A}{E}$ and transformation $\left(\frac{1+R}{1+r}\right)^T - 1$, which increases in T. When leverage is limited by regulation (the capital requirement implies that $A/E \leq 1/k$) shareholders can increase their expected ROE by increasing T, hence the motivation for maturity transformation.

Suppose now that a grain of sand is introduced into the wheels of the perfect markets world: at each period, there is a probability λ that the bank is not able to refinance, either because financial markets are dysfunctional or because investors are suspicious about the real quality of the bank's assets. The probability that the bank will be able to survive the T cycles of refinancing is $(1-\lambda)^T$. If instead the bank has a (re)financing problem at one of the dates $t=0,\dots T-1$, then shareholders lose everything. The expected return on equity becomes

$$ROE = \frac{A}{E}\left[\left\{(1-\lambda)\left(\frac{1+R}{1+r}\right)\right\}^T - 1\right].$$

This is negative if $(1-\lambda)\left(\frac{1+R}{1+r}\right) < 1$, which is equivalent to $\lambda > \frac{R-r}{1+R}$.

Numerical Example: $R = 5\%$ $\quad R-r = 0.5\%$.

The expected return on equity is negative if

$$\lambda > \frac{R-r}{1+R} = \frac{0.5}{105} \simeq 0.474\%.$$

Thus, a very small probability of market dysfunctionality is sufficient to make the Northern Rock strategy unprofitable.

7.5.2 Fallacy 2: Leverage Does not Matter

Consider a firm with cash reserves m, who invests I in some assets that will generate a stream of future earnings. When financial markets are perfect, the value of the investments is measured by the risk neutral (present) value of these future earnings, which we denote by A. We assume that the investment is profitable: A exceeds the amount invested, I. Shareholders want to determine the optimal way to finance

this investment. If $m > I$, they can fully self-finance their investment and distribute the excess cash $(m-I)$ in the form of dividends. In this case, shareholder value[7] (with self-financing) is given by:

$$SV_0 = m - I + A. \tag{7.2}$$

An alternative solution is to borrow D against the promise to repay a certain stream of future coupons and repayments (financial expenses), which allows us to distribute a larger dividend $d = m + D - I$ at $t=0$. However, the ex-dividend value of equity is reduced to some lower value, E, and total shareholder value becomes (with debt financing):

$$SV_1 = m + D - I + E. \tag{7.3}$$

A natural question is: Which type of financing (self-finance or debt) or more generally what financial structure (i.e., combination of debt and equity) maximizes shareholder value?

To compare SV_1 with the shareholder value SV_0 obtained when $D=0$ (complete self-finance), we have to evaluate E. This can be done easily when financial markets are perfect (efficient and complete). In this case, E is given by the risk neutral present value of future dividends:

$$E = \mathcal{E}_Q[\text{Discounted Future Dividends}], \tag{7.4}$$

where $\mathcal{E}_Q$ represents the expectation operator under the risk neutral probability Q. Similarly, the amount D borrowed at $t=0$ is equal to the risk neutral expectation of future coupons and debt repayments (financial expenses):

$$D = \mathcal{E}_Q[\text{Discounted Future Financial Expenses}]. \tag{7.5}$$

In the absence of frictions, the sum of the two terms between brackets must be equal to future earnings,[8] thus:

$$E + D = \mathcal{E}_Q[\text{Discounted Future Earnings}] = A.$$

Comparing (7.2) and (7.3) we see that shareholder value is the same under self-financing (SV_0) or under debt financing (SV_1)!

Thus, we have established the following, paradoxical result:

Result 7.1. *(The Modigliani-Miller Theorem) When financial markets are perfect, leverage has no impact on shareholder value.*

This counterintuitive result is called the Modigliani-Miller theorem. The famous economists Modigliani and Miller, who were the first to prove it, were obviously dissatisfied with it. They showed that when corporate taxes are taken into account, leverage matters. The reason is that in most countries, interest expenses are tax deductible.

Thus, corporations have an incentive to increase leverage to minimize taxes. However, increasing leverage also increases the probability that the firm goes bankrupt, and bankruptcy costs are non-negligible. Using the same methodology as in the previous section, we can write a new formula for shareholder value, which incorporates taxes and bankruptcy costs:

$$SV = SV_0 - \mathcal{E}_Q[\text{Discounted Future Taxes}] - \mathcal{E}_Q[\text{Discounted Future Bankruptcy Costs}],$$

where SV_0 represents the shareholder value computed above, in a world without taxes nor bankruptcy costs.

Generally speaking, the trade-off between tax optimization and bankruptcy costs implies an optimal level of debt. This optimal level is obtained by minimizing the expected sum of taxes and bankruptcy costs.

7.5.3 Fallacy 3: Risk Management is Useless

An equally paradoxical result can be deduced from the perfect markets assumption: Risk transfers can only **reduce** shareholder value at least for a large firm that is held by a large number of diversified shareholders.

As an illustration, consider the following (fictitious) example. Companies A and B sell sports items, such as soccer jerseys of national teams. Company A specializes in European teams, whereas company B only sells the jerseys of the Brazilian national team. Given that sales increase dramatically for the national team that wins the FIFA World Cup final, company B considers insuring against the risk that Brazil does not win the next World Cup final. This makes sense if company B is held by its manager. It is a nonsense if companies A and B are publicly listed. By holding equal[9] numbers of shares of companies A and B, each investor can costlessly hedge the risk: the gains on the winner will exactly offset the losses of the loser.

However, note that as we already pointed out, the manager of the firm may have different views than shareholders. In particular, he may attach more weight to downside risk than shareholders. Indeed, if the firm makes unexpected profits, this will essentially benefit shareholders. By contrast, the manager incurs the risk of having to find a new job if the firm goes bust or if he is fired after unexpected losses made by the firm. The risk cannot be perfectly insured by market instruments that cover this risk. Thus, the manager has a strict preference for self-financing investments and limiting leverage to a minimum (and more generally for financial strategies that reduce downside risks) to minimize the probability of bankruptcy of the firm.

The generality of the uselessness of risk management within the perfect markets world can be established very easily. Any risk transfer operation (through a market instrument like futures, options,... or an insurance contract) has a zero risk neutral present value (at best: it will in fact be negative if intermediation costs or market power are present). Thus, using the Modigliani-Miller formula first before the risk

transfer

$$\text{Shareholder Value} = \mathcal{E}_Q[\text{Discounted Future Earnings}] - \mathcal{E}_Q[\text{Discounted Financial Expenses}]$$

and then after the risk transfer:

$$\text{Shareholder Value} = \mathcal{E}_Q[\text{Discounted Future Earnings}] - \mathcal{E}_Q[\text{Discounted Financial Expenses}] + \mathcal{E}_Q[\text{Risk Transfer Payments}],$$

we see that the two values coincide so that risk management[10] is useless.